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Reactions of Mill Building Frames

Civil Engineering

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REACTIONS OF MILL BUILDING FRAMES

BY

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AND
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THESIS

FOR THE

DEGREE OF BACHELOR OF SCIENCE

IN

CIVIL ENGINEERING

COLLEGE OF ENGINEERING

UNIVERSITY OF ILLINOIS

1917

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UNIVERSITY OF ILLINOIS

May 25, 1917

THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

THOMAS ELMER STOCKDALE and WALTER AUGUST HIMMELREICHER

ENTITLED REACTION OF MILL BUILDING FRAMES

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF BACHELOR OF SCIENCE

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Instructor in Charge

APPROVED: F. H. Newell

HEAD OF DEPARTMENT OF CIVIL ENGINEERING

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REACTIONS OF STEEL MILL BUILDING FRAMES.

I. INTRODUCTION

Certain assumptions must be made in the design of all engineering structures in order that safety and economy be secured. It is probable that this will always be true to a greater or less extent, but each year sees new methods devised which do away with one assumption or another, or make possible more rational assumptions. It is the purpose of this thesis to make a beginning toward collecting data so that at least one of the assumptions commonly made in steel mill building design may be more rationally and intelligently made.

CLASSIFICATION OF MILL STRUCTURES

Mill buildings, for the purpose of this discussion, are classified according to the condition of the ends of the columns, i.e., whether they are pinned or fixed. This discussion will be limited to bents the columns of which are considered pinned at their bases.

THE PROBLEM

In order that the stresses in the various members of a mill building bent be determined, it is necessary that the reactions at the bases of the columns be known, both in magnitude and direction. The vertical components of these reactions may be easily found by statics, but the magnitude of the horizontal components must be found in some other way.

Usually they are assumed to be equal, and each equal to one-half of the total horizontal load on the structure. It is proposed to investigate the validity of this assumption in a few cases.

II. THEORY

The equations of statics may be supplimented by a consideration of the elastic properties of the structure so that the problem of determining the reactions becomes determinate. As far as the writers know this has never been done to structures of the mill building type. The fundamental theory on which this investigation is based is the application of Maxwell's Law of Reciprocal Deflections to a combination of the truss and beam such as the ordinary mill building bent. That this application is possible has been proved,* and will not be further discussed here.

Consider a typical mill building bent, as Fig. 1. The point A is considered fixed in position, while the point J is on rollers and is free to move horizontally. Any load on the structure, as the load W at point E, causes a certain horizontal displacement of the point J equal, say, to d_1. A unit load applied horizontally at J will displace the point J an amount equal to, say, d_2. Then, according to the theory of elasticity, the force necessary to push the point J from its displaced position due to the load W, to its original position is d_1 / d_2.

* Professor C.A.Ellis, in an article not yet published.

According to Maxwell's Law of Reciprocal Deflections, if a force W acting in any direction (vertical) at E causes a displacement d_1 in any direction (horizontal) of the point J, then the force W acting horizontally at J will cause a vertical displacement of the point E equal to d_1. This quantity, d_1, will be noted to be equal to W times the displacement of the point E due to a 1 lb. force acting horizontally at J. It is necessary, then, to construct only one Williot diagram, that showing the displacements of the various panel points due to the 1 lb. force acting horizontally at J. If

d = displacement of E in the direction of W, due to a 1 lb. force acting horizontally at J,

$$H = \frac{W \times d}{d_2}.$$

Hence, in order to find the total horizontal component of the reaction at the base of the lee column, it is only necessary to find H in the above formula for the load at each panel point, and to add these quantities.

METHODS OF DETERMINING DISPLACEMENTS

The flexure in the columns may be most easily found by the area-moment method. The displacements in the truss may be found either algebraically or graphically. The graphic solution, by means of the Williot diagram, will be used.

III. PROBLEM 1.

A typical mill building bent, as shown in Fig. 2, will be considered. The truss is designed according to the method recommended by Fleming, for a load of 40 lb. per sq. ft. on a horizontal projection. The columns are designed to withstand a horizontal wind load of 30 lb. per sq. ft. The sections thus obtained for the various members are listed in column 3, Table 1.

The point Q is considered fixed in position, although not capable of resisting moment. The point P is considered as on rollers, being free to move horizontally when the structure is loaded. The horizontal displacement of the point P, due to a 1 lb. force acting horizontally at P, is first determined.

DISPLACEMENTS IN TRUSS

The horizontal force of 1 lb. at P necessitates a like force at Q, and these develop horizontal reactions on the truss at A and B, J and H, as shown in Fig. 4. The stresses in the truss due to this loading are determined by a stress diagram, and are tabulated in column 6, Table 1.

The strains (elongations) of the members are now computed and listed in column 7, Table 1, and the Williot diagram, Fig. 6, is drawn. Fig. 7 shows the bent before and after loading, the displacements being of course greatly exaggerated.

DEFLECTION OF COLUMNS

The columns are loaded as shown in Fig. 5 a. The shear and moment diagrams, Figs. 5 b and 5 c, are drawn and the displacements determined as follows:

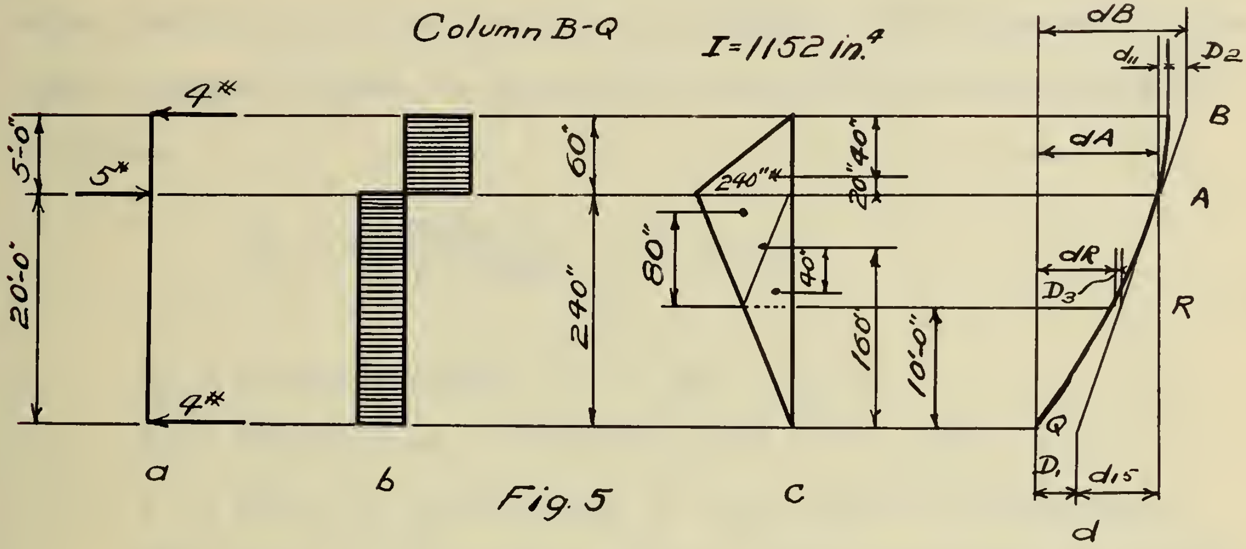

Fig. 5

$$D_2 = \frac{1}{EI}\left(\frac{240\times60\times40}{2}\right) = \frac{288000}{1152\ E} = \frac{250}{E} \text{ inches.}$$

$$D_1 = \frac{1}{EI}\left(\frac{240\times240\times160}{2}\right) = \frac{4606000}{1152\ E} = \frac{4000}{E} \text{ inches.}$$

$$D_3 = \frac{1}{EI}\left(\frac{240\times120\times80}{2} + \frac{120\times120\times40}{2}\right) = \frac{1440000}{1152\ E} = \frac{1250}{E} \text{ inches.}$$

From the Williot diagram, $d_{11} = \dfrac{1670}{E}$

$$d_{15} = 4(d_{11} + D_2) = \frac{7480}{E}$$

$$dA = D_1 + d_{15} = \frac{11480}{E}$$

$$dB = dA + d_{11} = \frac{13100}{E}$$

$$dR = dA - \frac{1}{2}d_{15} - D_3 = \frac{6490}{E}$$

The total displacement of the point P from its original position $= 2\ dA + d_{22} = \dfrac{22960}{E} + \dfrac{4400}{E} = \dfrac{27360}{E}$.

LOADING

The horizontal component of the reaction at P due to a typical loading is now computed. The structure is loaded with 30 lb. per sq. ft. on a horizontal projection, and a horizontal wind load of 30 lb. per sq. ft. is assumed. The component of the wind pressure normal to the roof is determined by the Duchemin formula,

$$P_n = P_v \frac{2 \sin A}{1 + \sin^2 A}, \text{ in which}$$

P_n = normal pressure in lb. per sq. ft.

P_v = pressure on a vertical plane in lb. per sq. ft.

A = angle of inclination of roof with the horizontal.

The loads at the various panel points are given in column 2, Table 2, and on Fig. 3. From the Williot diagram the displacements of the panel points in the direction of the loads, due to a 1 lb. horizontal force at P, are found, and these displacements are tabulated in column 3, Table 2. Column 5, Table 2, gives the total horizontal component of the reaction at P due to each load, and the summation of this column gives the total horizontal component of the reaction at P. In this case this quantity is 5480 lb., or 44.4 % of the total horizontal load on the truss.

IV. PROBLEM 2.

A bent of the type shown in Fig. 8 was next investigated. The manner of attack was similar to that of Problem 1, and only data and results need be recorded here. The data follow on pages 17,18, 30, 39, 40, and the results are tabulated in Tables 15 and 16.

V. PROBLEMS 3, 4, 5, 6, and 7.

It was noted that there was a radical difference between the results of Problems 1 and 2, as shown in columns 2 and 3 of Table 15. In trying to account for this discrepancy it was decided to try the effect of varying some of the factors involved, abd for this purpose a simple bent, as Fig. 12, was chosen.

The height of the bent was kept constant and the length varied. Bents of 10 ft., 20 ft., 30 ft., 40 ft., and 60 ft, span were investigated in a manner similar to that of Problems 1 and 2.

RESULTS DESIRED

The results to be obtained from the above investigation are as follows:

(1) Ratio of total horizontal components to total horizontal loads,

(2) Ratio of horizontal components due to horizontal loads to said horizontal loads,

(3) Ratio of horizontal components due to vertical loads to said vertical loads,

(4) How the percentage(of a single horizontal load)that is carried by each reaction varies with the height of the point of application,

(5) How the quantities (1), (2), (3), and (4) vary with the length, height, and shape of the bent. Only the effect of varying the length of one type of truss-bent will be determined here.

VI. RESULTS.

Results of the foregoing problems are shown in Tables 15 and 16, and are also shown graphically by the curves, Figs. 28, 29, 30, 31, and 32.

RATIO OF TOTAL HORIZONTAL COMPONENTS TO HORIZONTAL LOADS

It will be seen from Table 15 and also from the curve, Fig. 28, that for this particular shape of bent, and for this loading, the ratio of total horizontal components of the reactions to the total horizontal load is a variable quantity, the variation being a function of the length of span. For the ratios of length to height investigated, the variation for the lee reaction was from 41.5 % to 60.0 %.

The curve shows that the ordinary assumption that the horizontal components of the reactions are equal is true for only one value of the ratio of length to height.

It must be remembered that this curve, Fig. 28, is applicable only to bents of the type from which the data were derived, and for the loading as stated above. For this reason it is thought that the Ratios (2) and (3) will be of greater service than Ratio (1).

RATIO OF HORIZONTAL COMPONENTS DUE TO HORIZONTAL LOADS TO TOTAL HORIZONTAL LOADS.

That the effect of vertical loads is not inconsiderable is shown by the curves, Figs. 29 and 30. These show the relation between the ratio of length to height and the percentage of the horizontal and vertical loads that get into the horizontal component of each reaction. It is interesting that when the

effect of the vertical loads is neglected, the ratio of horizontal reaction to horizontal load becomes practically constant. This explains why there was such a radical difference in the results of Problems 1 and 2. The tendency of the vertical loads is to increase the lee reaction and to decrease the windward one. In the case referred to, this increase of the lee reaction was sufficient to make it considerably greater than the windward one. However, this is an extreme case, as in ordinary practice bents of the proportions of the one in Problem 2 will seldom, if ever, be used.

RATIO OF HORIZONTAL COMPONENTS DUE TO VERTICAL LOADS TO VERTICAL LOADS

As was to be expected, the effect of vertical loads on the horizontal components of the reactions is greater the greater the ratio of length to height. This is shown in Fig. 30.

VARIATION OF PERCENTAGE OF HORIZONTAL COMPONENT CARRIED BY EACH REACTION WITH THE HEIGHT OF THE POINT OF APPLICATION

This quantity is shown in the curves Figs. 31 and 32. It is seen to be independent of the length of the bent (Fig. 31). Also, the higher the point of application the nearer equal are the amounts getting into the horizontal components of the reactions.

From Fig. 32 it would appear that as the height of the point of application increases, the percentage reaching thelee reaction rapidly increases until the junction of the column with the truss is reached. Above this point the percentage remains practically constant at about 50 %.

GENERAL DISCUSSION OF CURVES

The data pertaining to Problems 3, 4, 5, 6, and 7 are plotted on the curves in black while that of Problems 1 and 2 are in red and blue, respectively. It will be noted that although they represent trusses of radically different shapes and proportions, they follow closely the results of the simple bent of Problems 3, 4, 5, 6, and 7. It would seem from the data at hand that the effect of the shape of the bent is not as important as that of the ratio of length to height. However, no general conclusions can be drawn without a much larger amount of data than are now available.

APPLICATION OF RESULTS TO DESIGN

After a number of sets of curves such as are presented here, for several common types of bents have been drawn, the proceedure for design would be either of two ways: (1) by the use of the curve Fig. 28, or (2) by the use of the curves Figs. 29 and 30. These methods will now be outlined.

(1) For any truss, determine the ratio of length to height. Pick a curve similar to Fig. 28 for a type of bent approximating the bent to be designed. Enter the diagram at the proper ratio of length to height, follow up to the curve and then go horizontally to the border and read the proper percentage of the total horizontal load that is carried by the lee (or windward, depending on which curve is used) reaction.Having determined this, it remains to be seen whether the maximum moment in the column is caused by say 40 % of the horizontal load, on the lee side, or by the 60 % on the windward side, the moment on the windward side being reduced, of course, by the moment of the

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horizontal loads on the column.

(2) The second method, which is perhaps somewhat more accurate than the first, if a little longer, is as follows:

Obtain curves similar to Figs. 29 and 30 for a type of bent similar to the one to be designed. Using each curve in a manner similar to that just explained for Fig. 28, determine the percentage of the horizontal loads, and also the percentage of the vertical loads that get into the horizontal components of the reactions. These two quantities are combined, after which the proceedure is the same as for the first method.

It is believed that the methods just outlined will give more accurate designs than the methods now in vogue.

VII. FIGURES

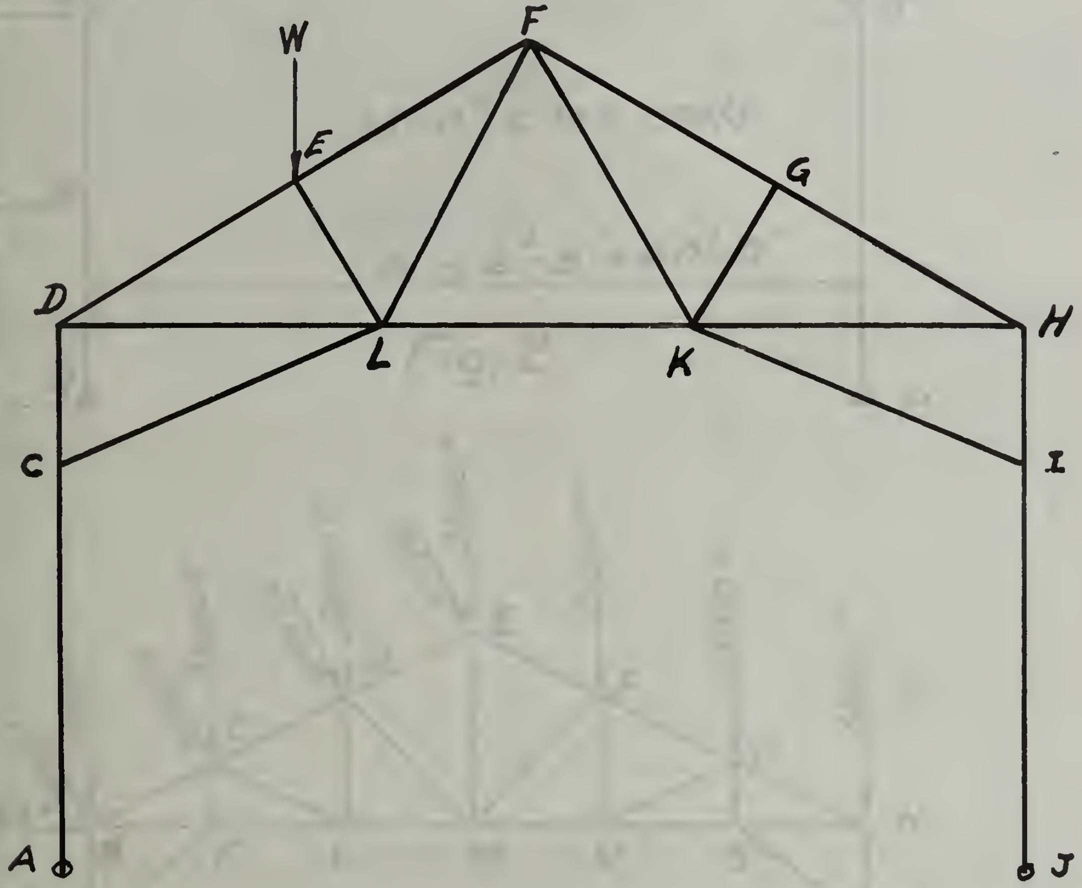


Fig. 1.

PROBLEM 1

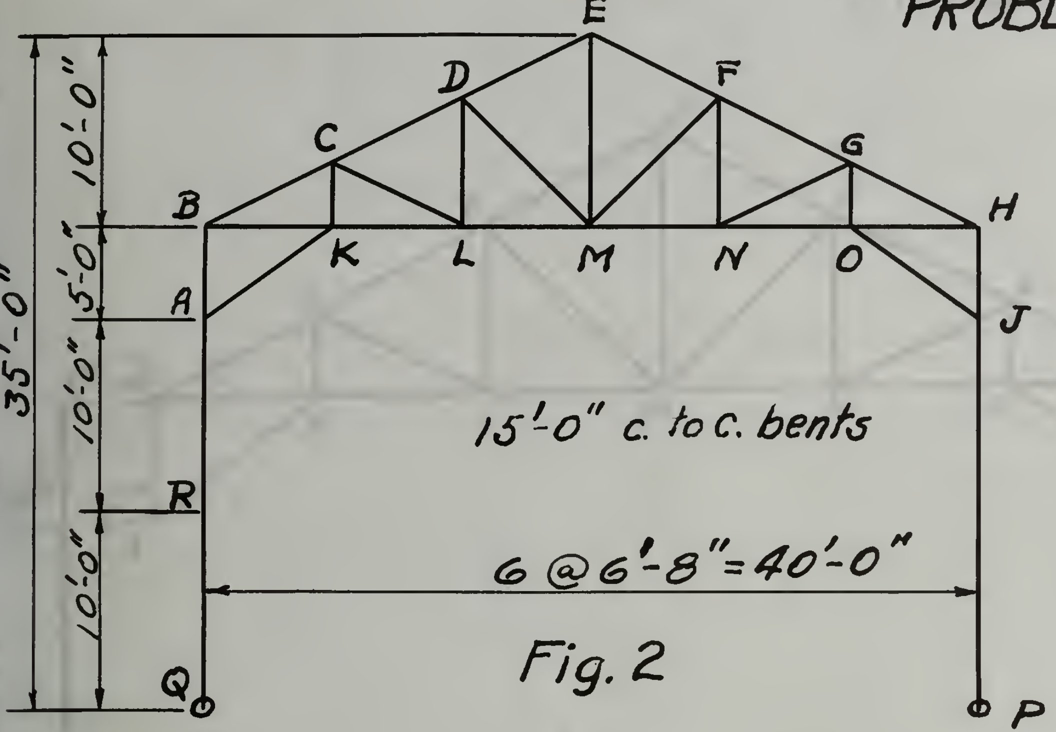


Fig. 2

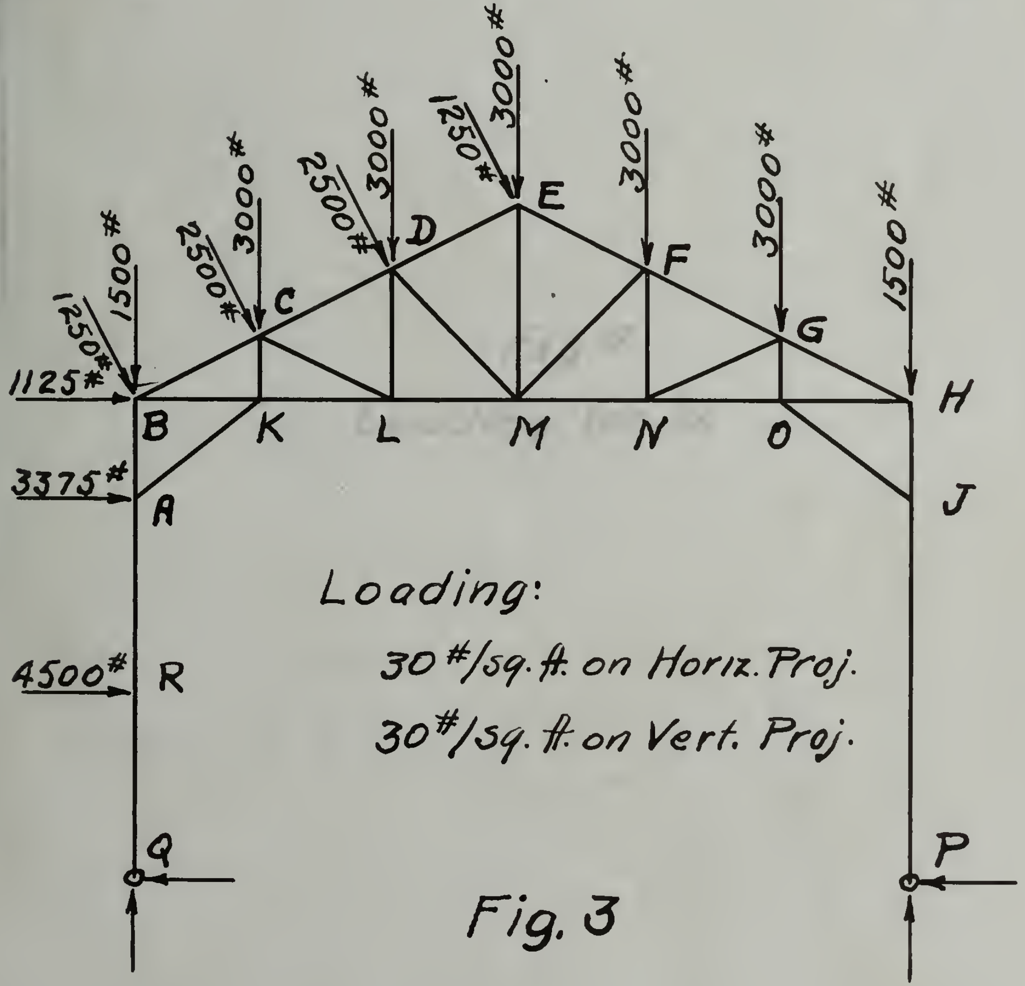


Fig. 3

PROBLEM 1

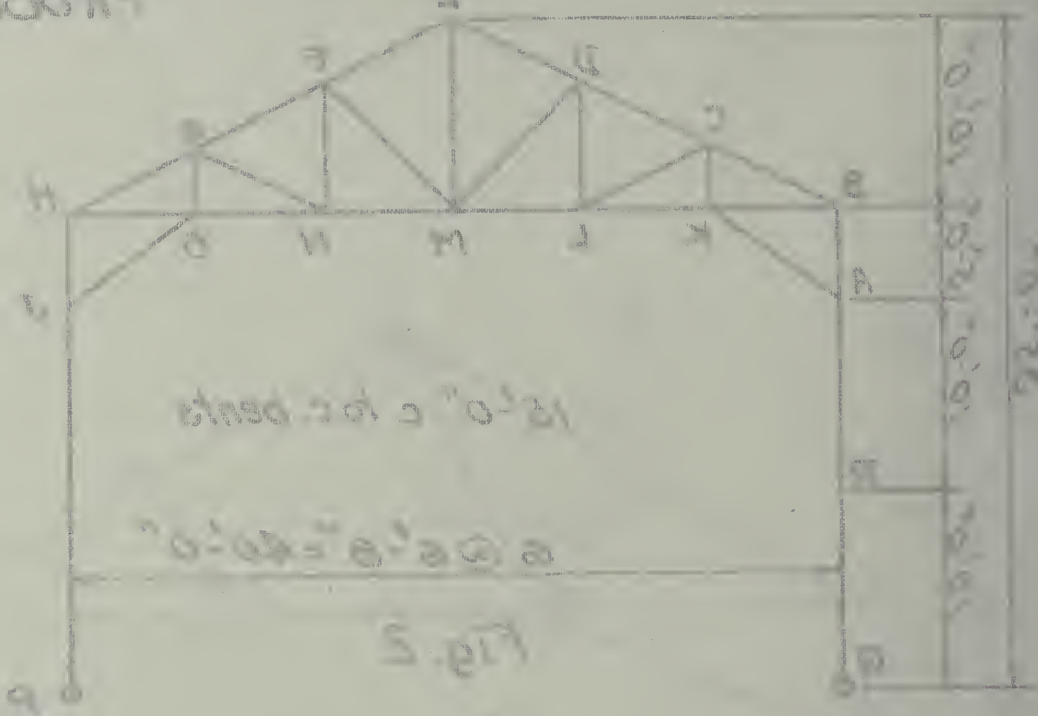


Fig. 2

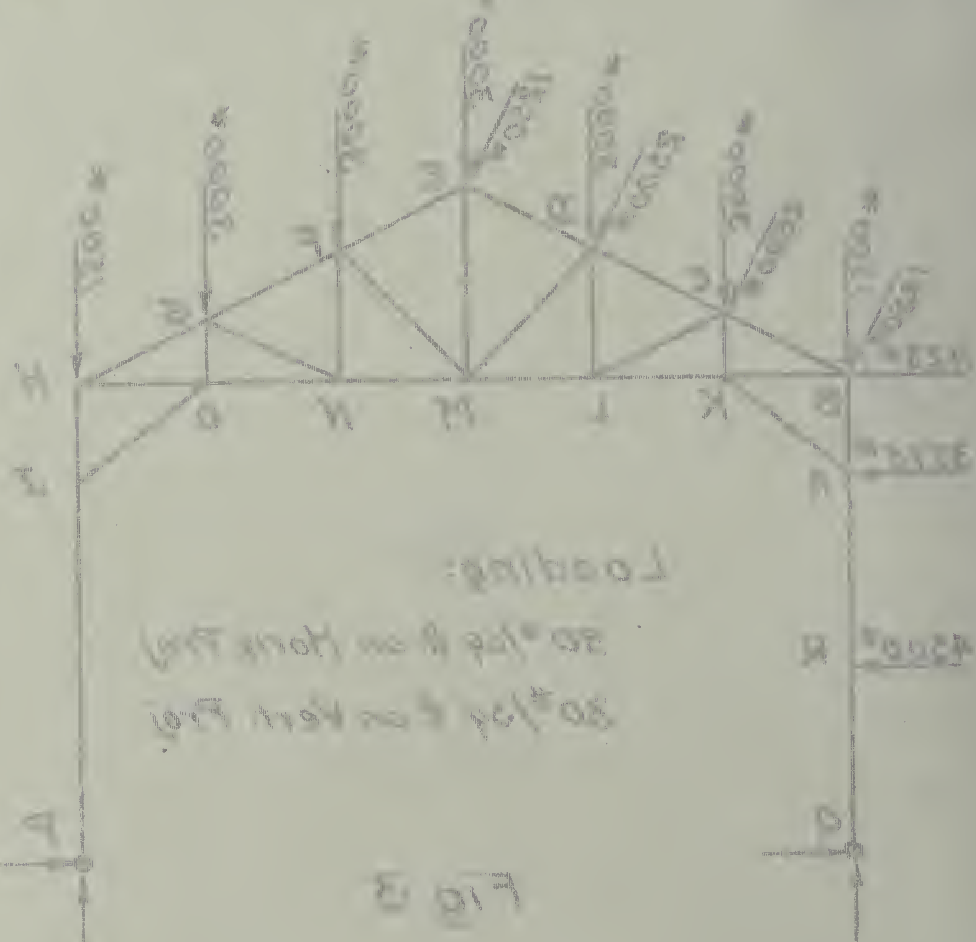


Fig. 3

PROBLEM 1

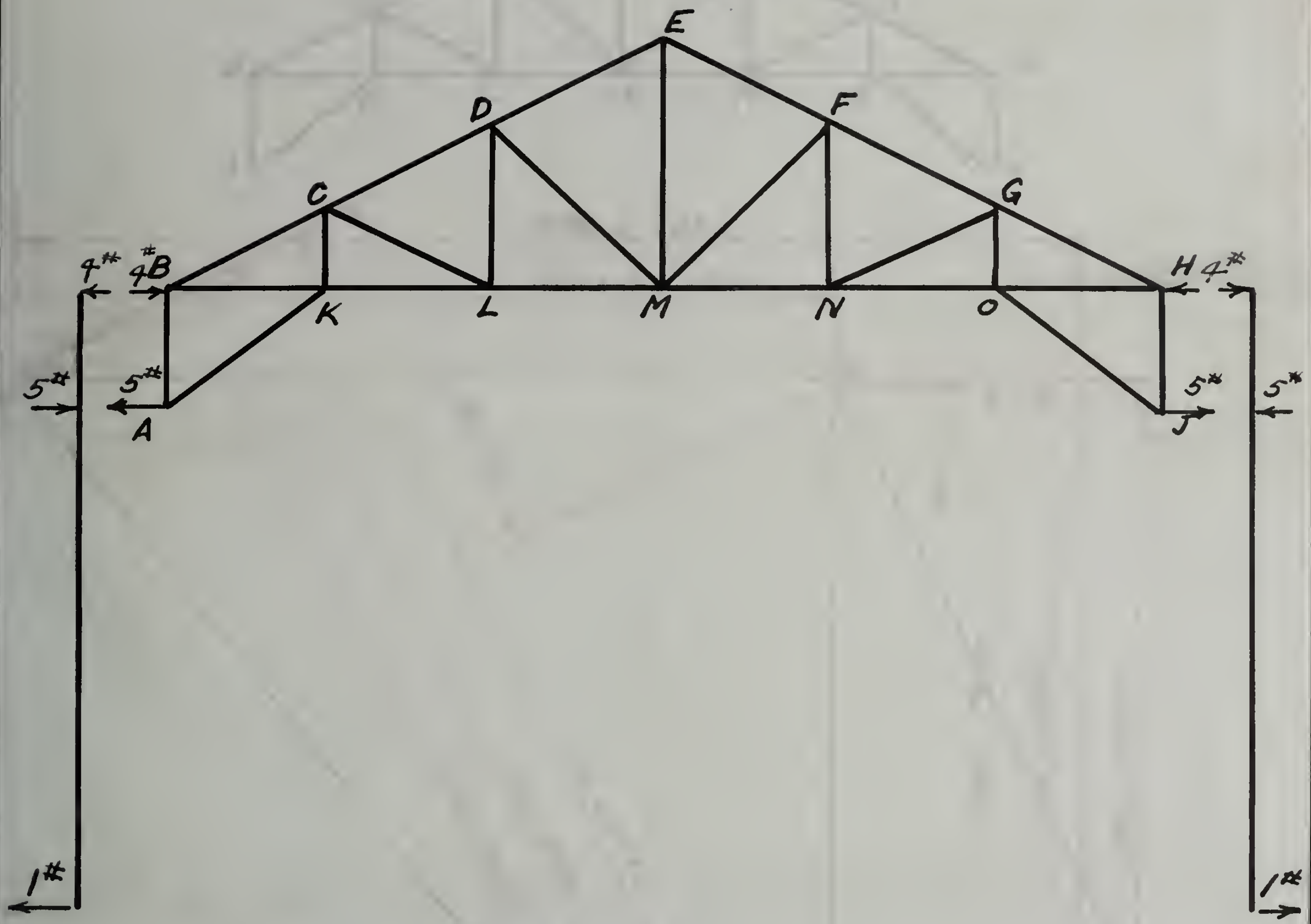


Fig. 4
Loading for U_1

PROBLEM 1

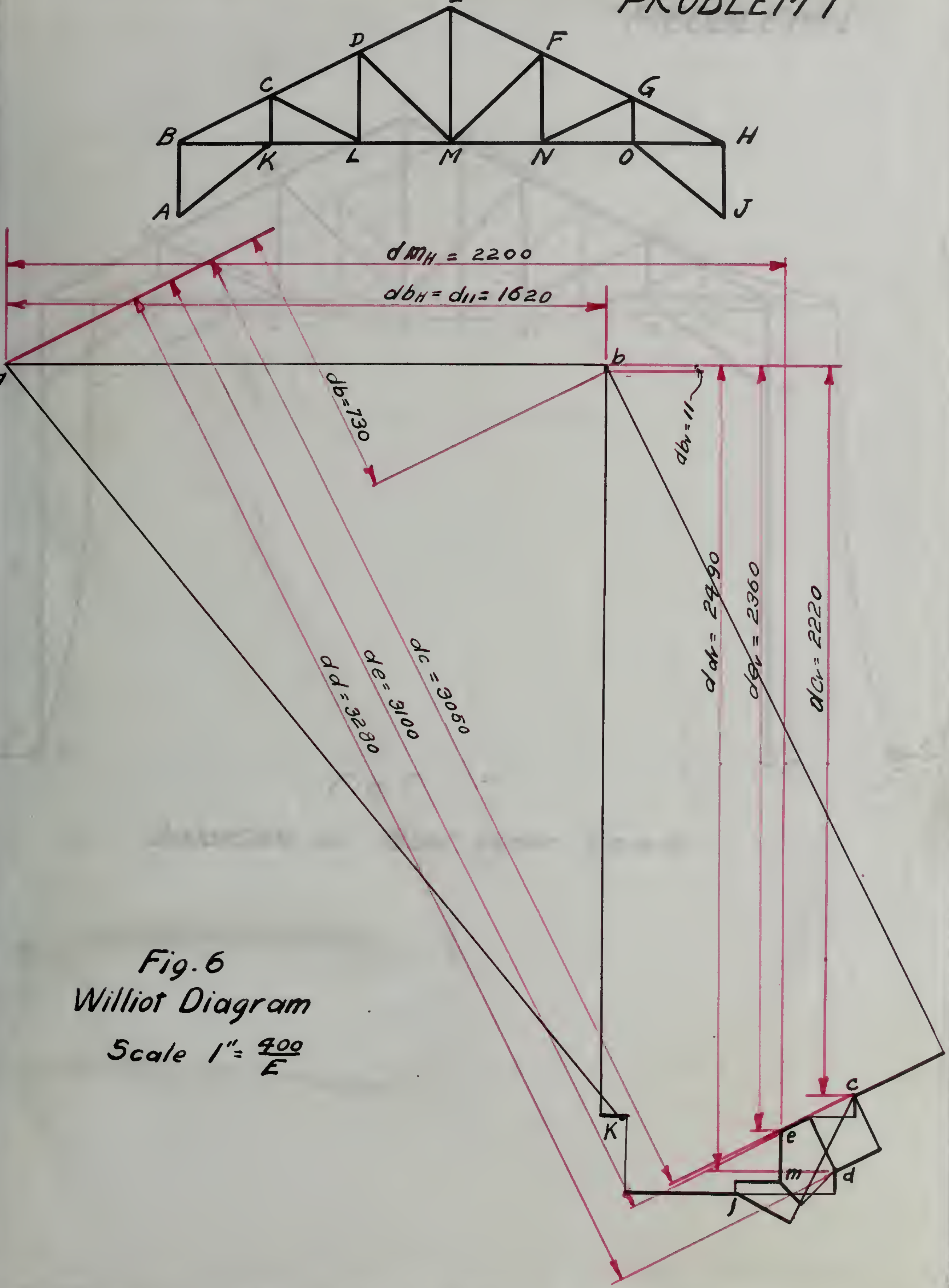
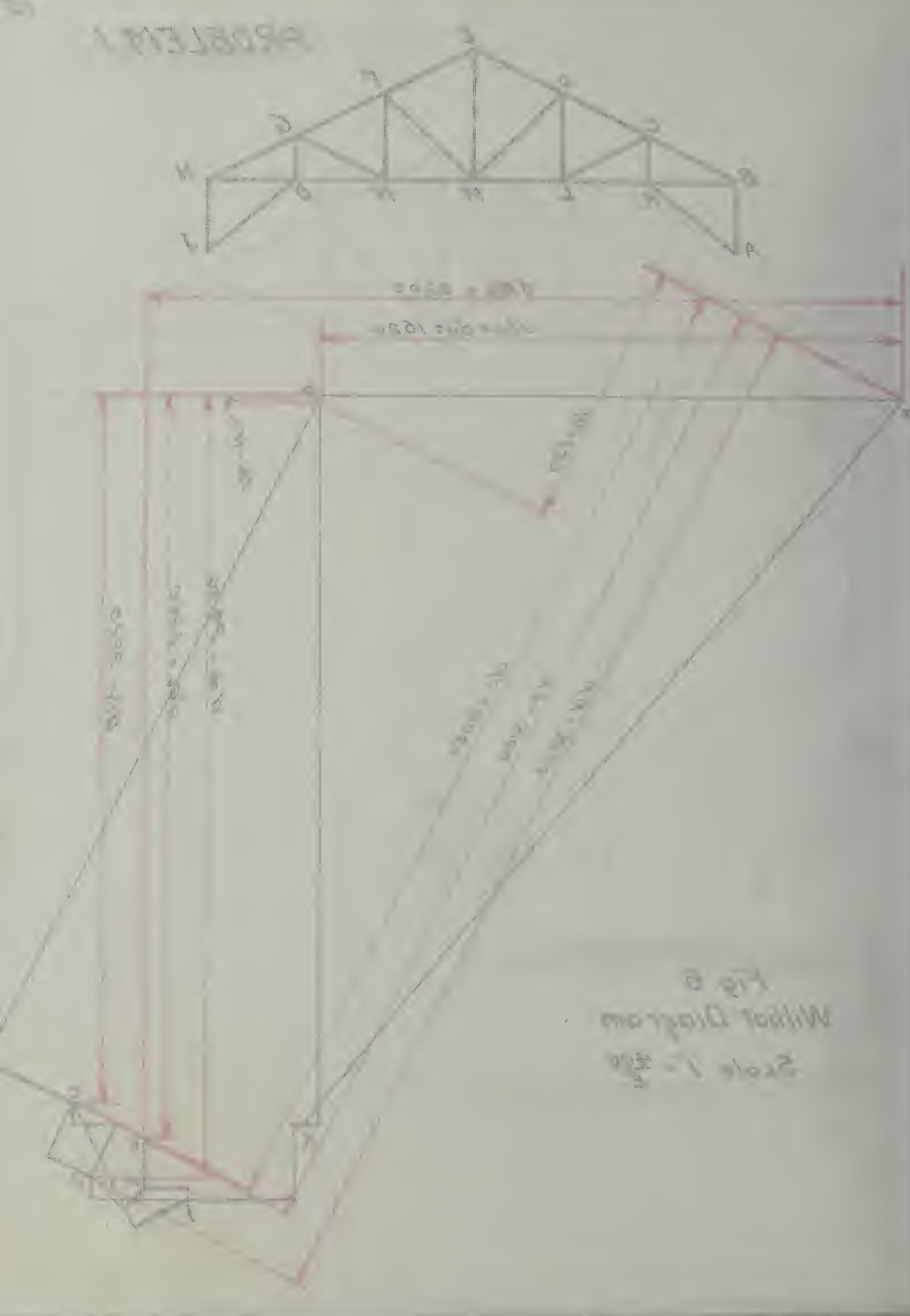


Fig. 6
Williot Diagram
Scale 1" = $\frac{400}{E}$



PROBLEM I

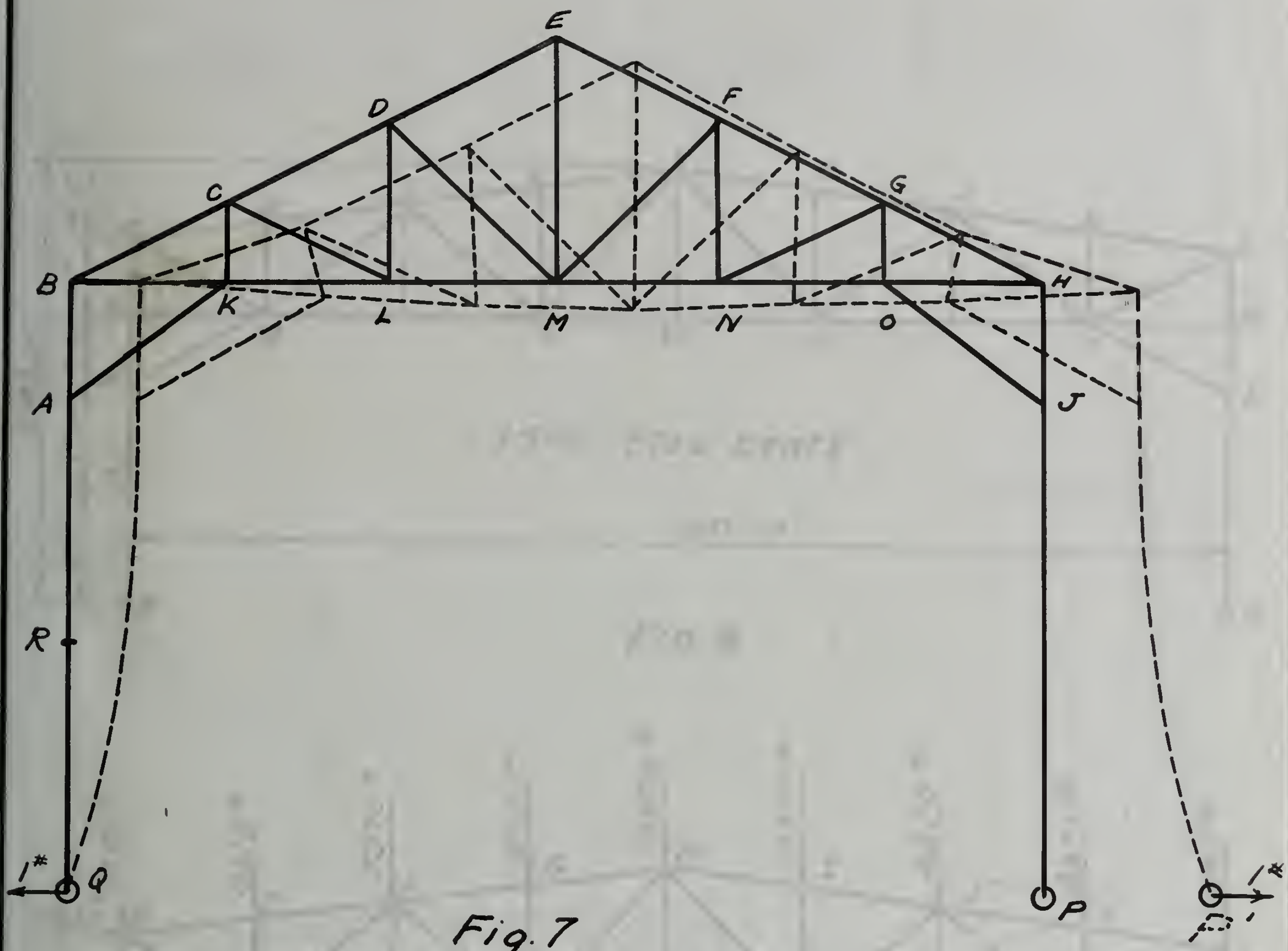


Fig. 7

Distortion of Bent under Load

PROBLEM 2

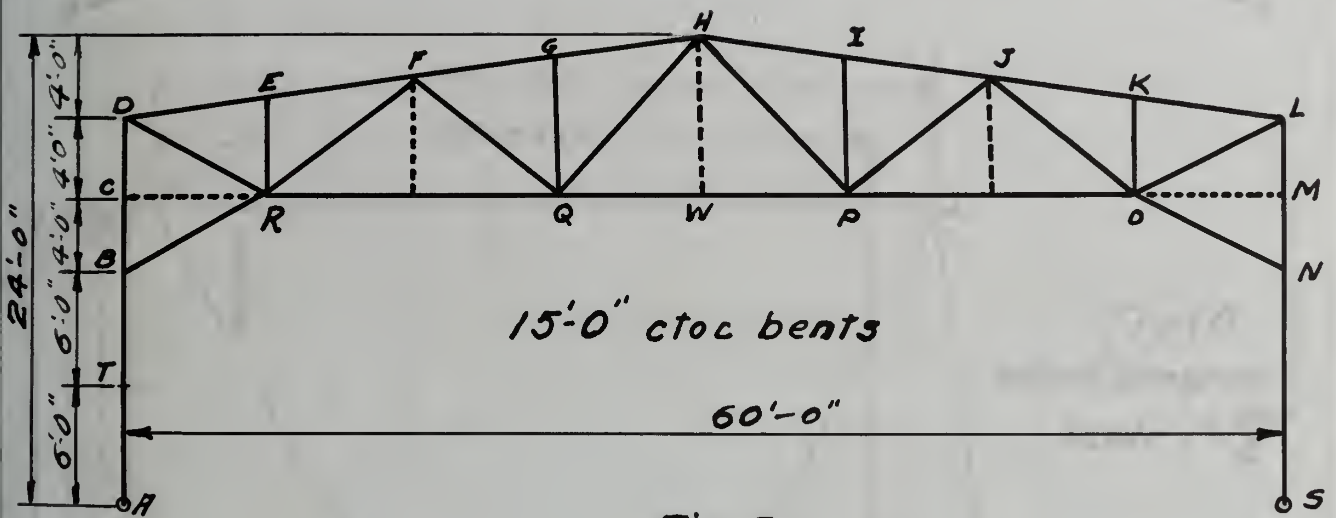


Fig. 8

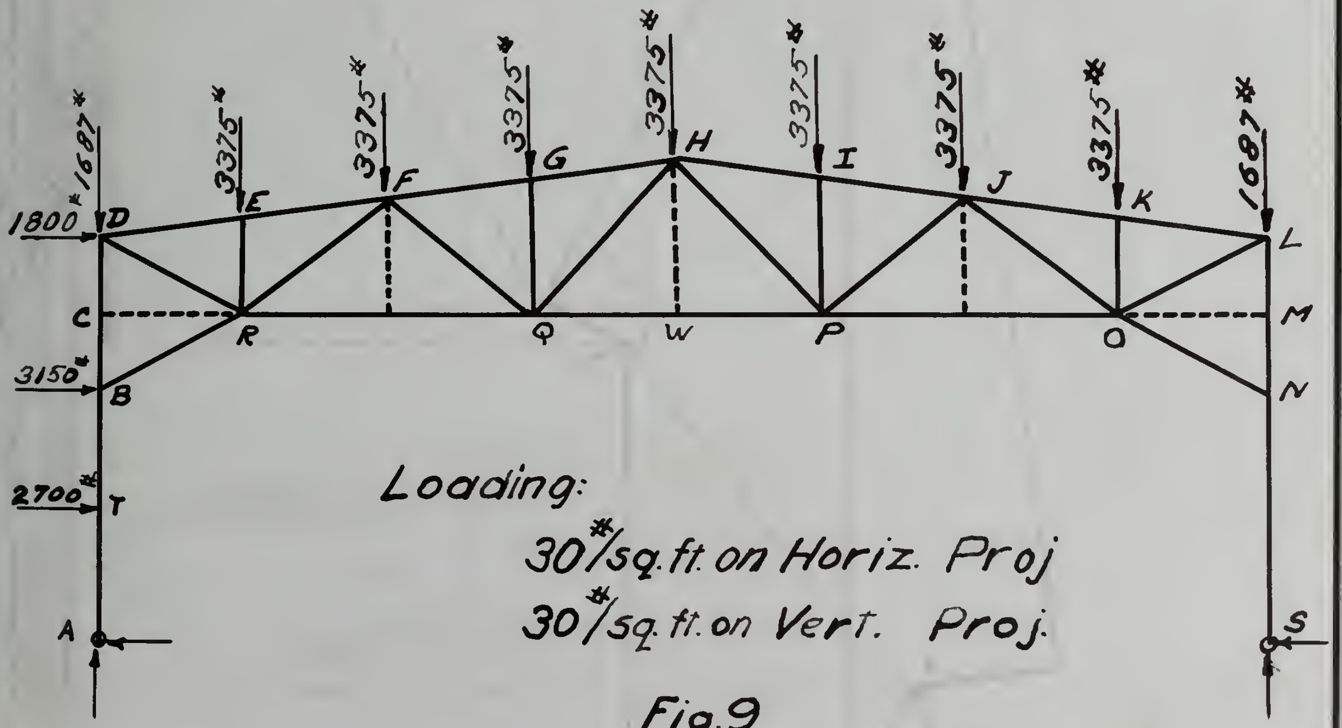
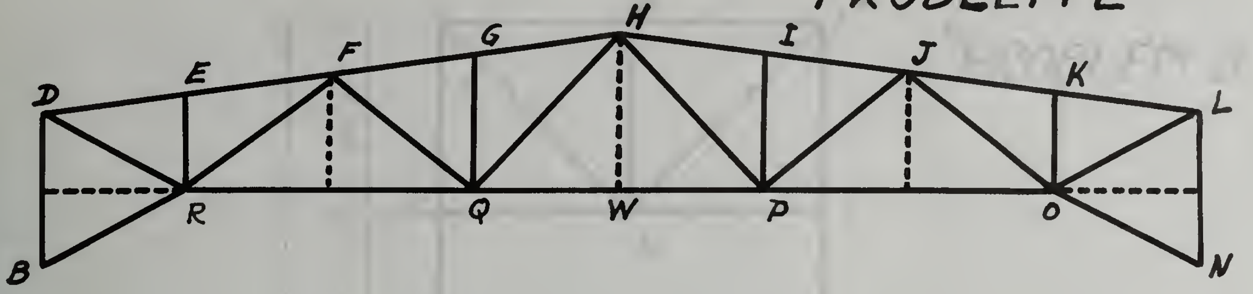


Fig. 9

PROBLEM 2



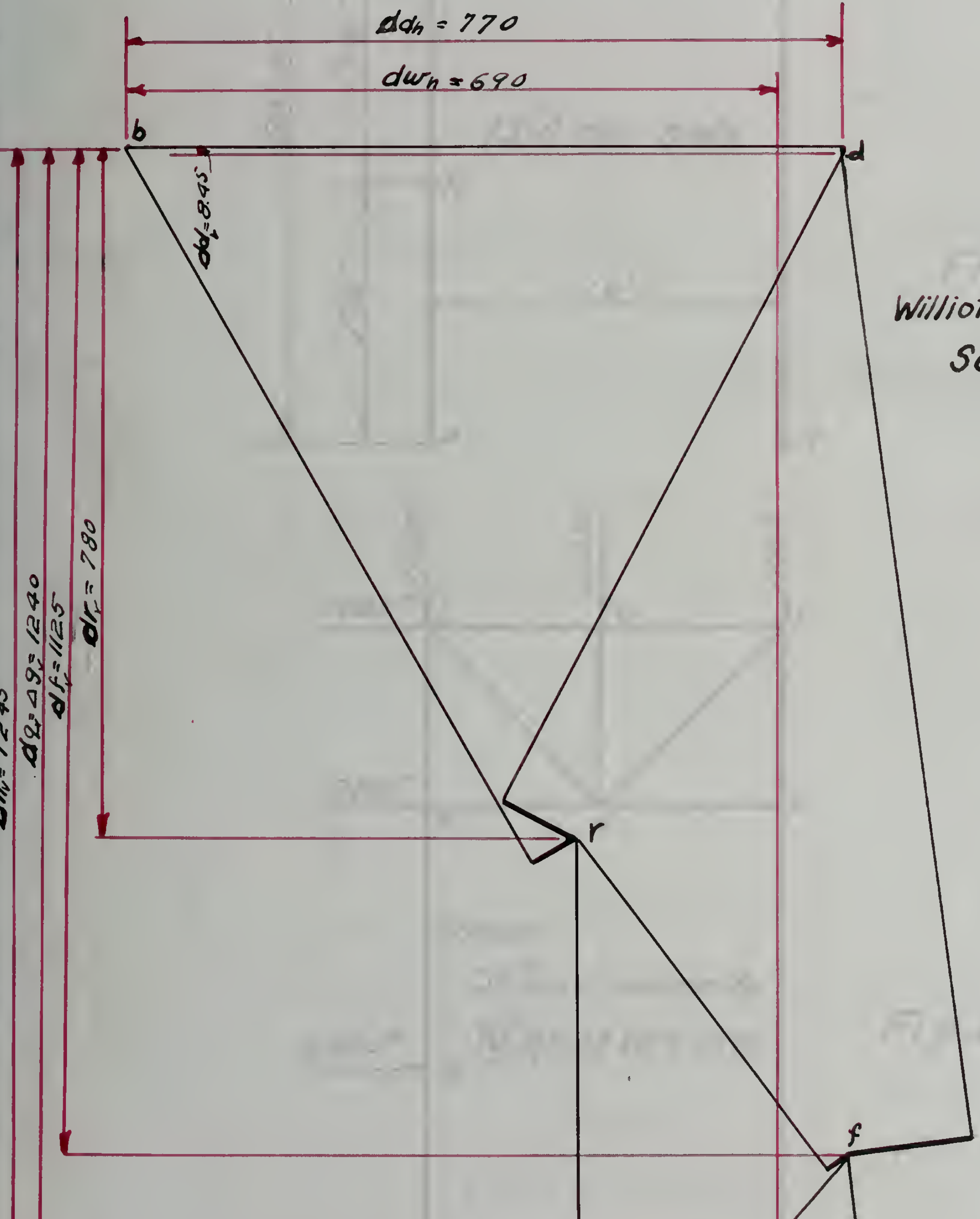
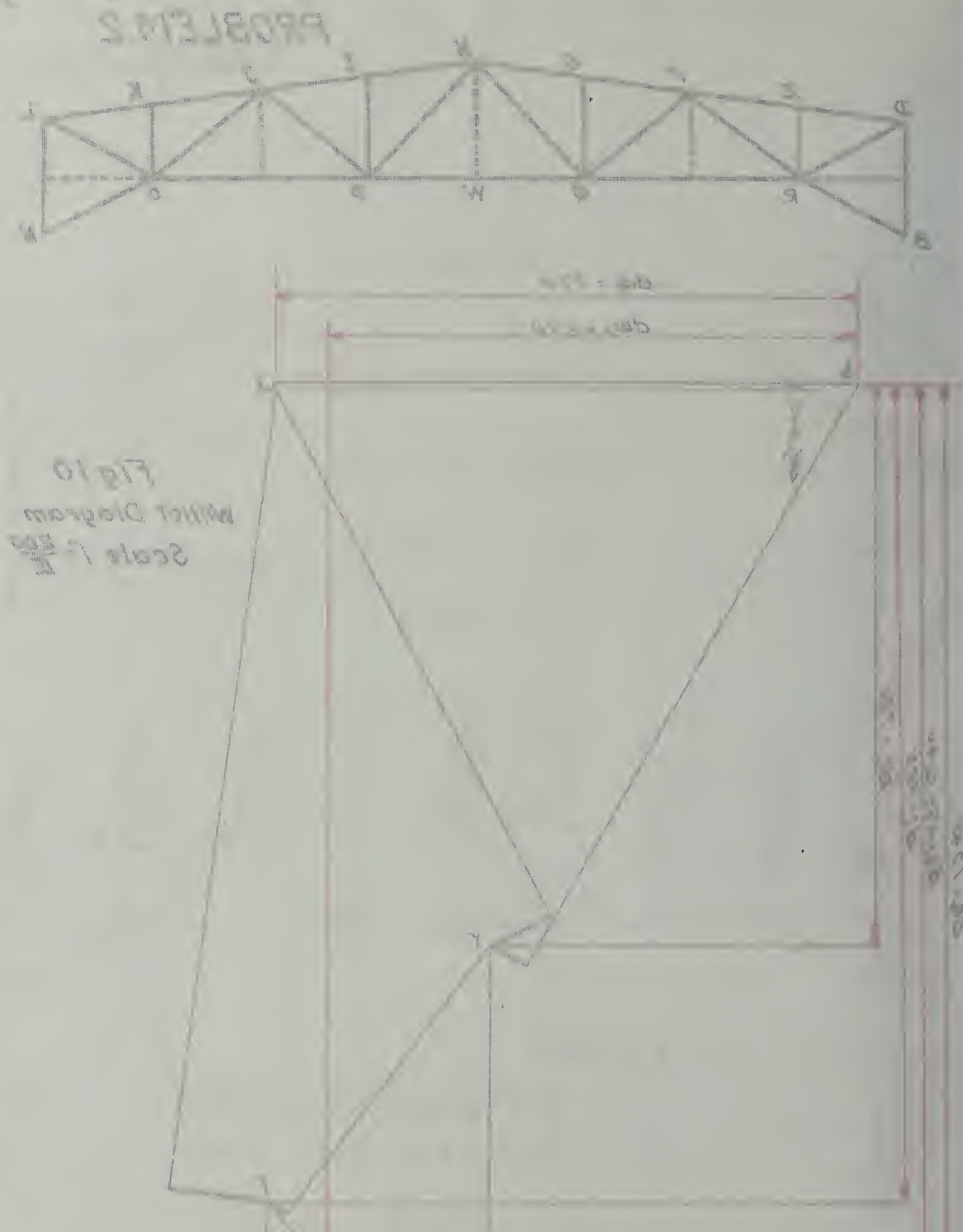


Fig. 10
Williot Diagram
Scale 1" = $\frac{200}{E}$


PROBLEM 2
Fig 10
Miller Diagram

PROBLEM 3

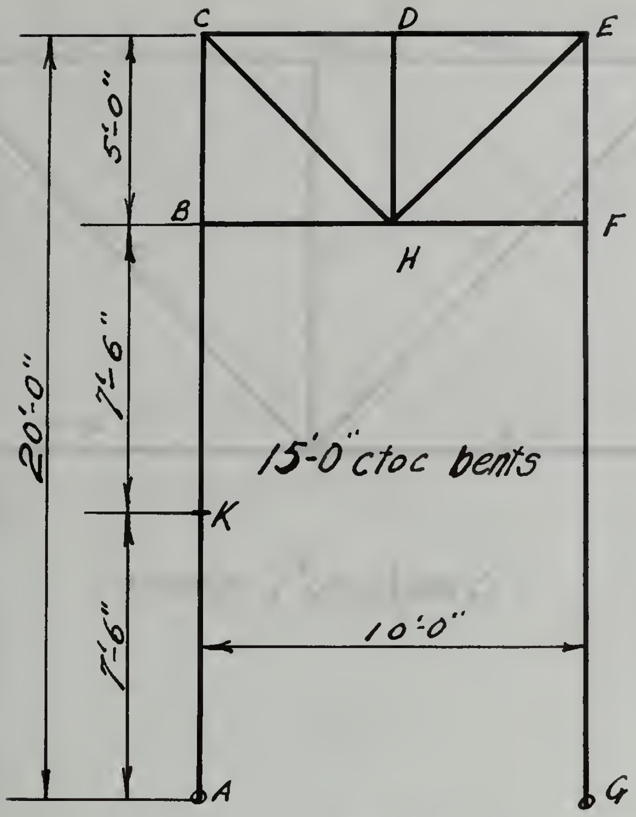


Fig. 12

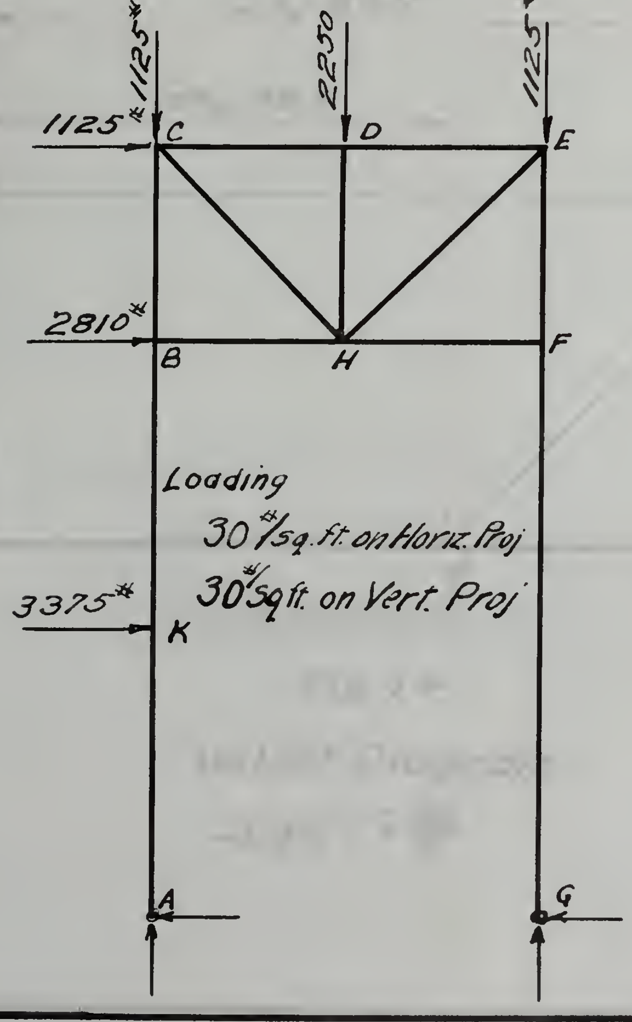
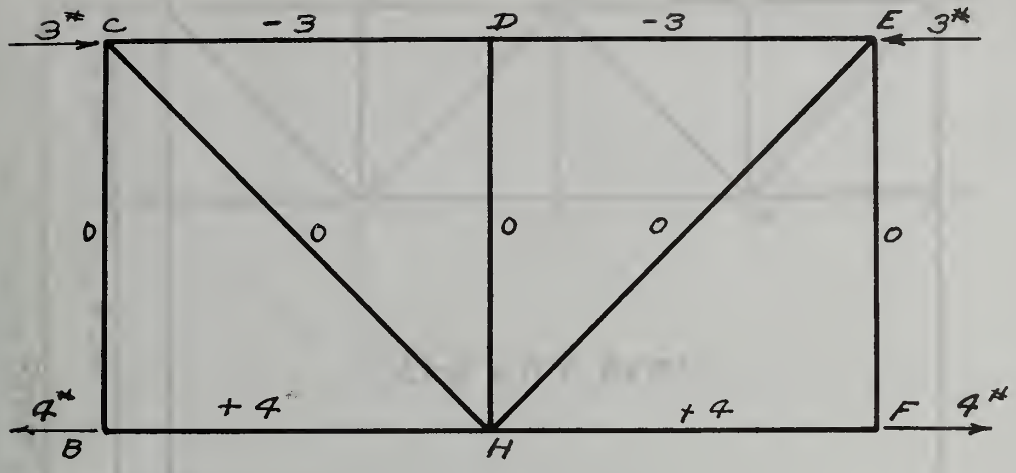


Fig. 13

PROBLEM 3



Loading: 1* at A and G

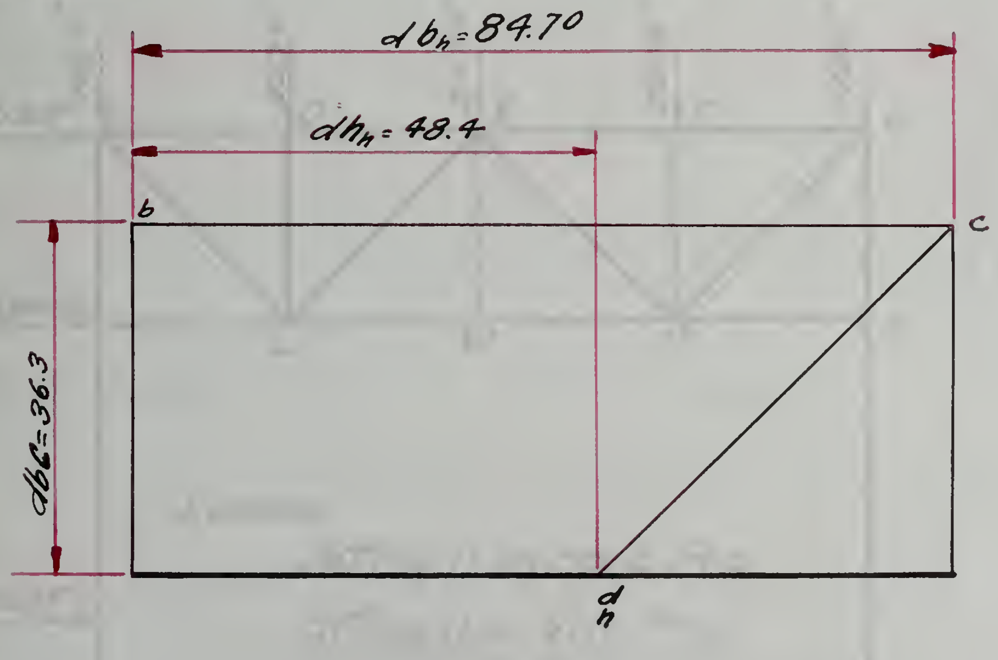


Fig. 14

Williot Diagram

Scale $1'' = \frac{20}{E}$

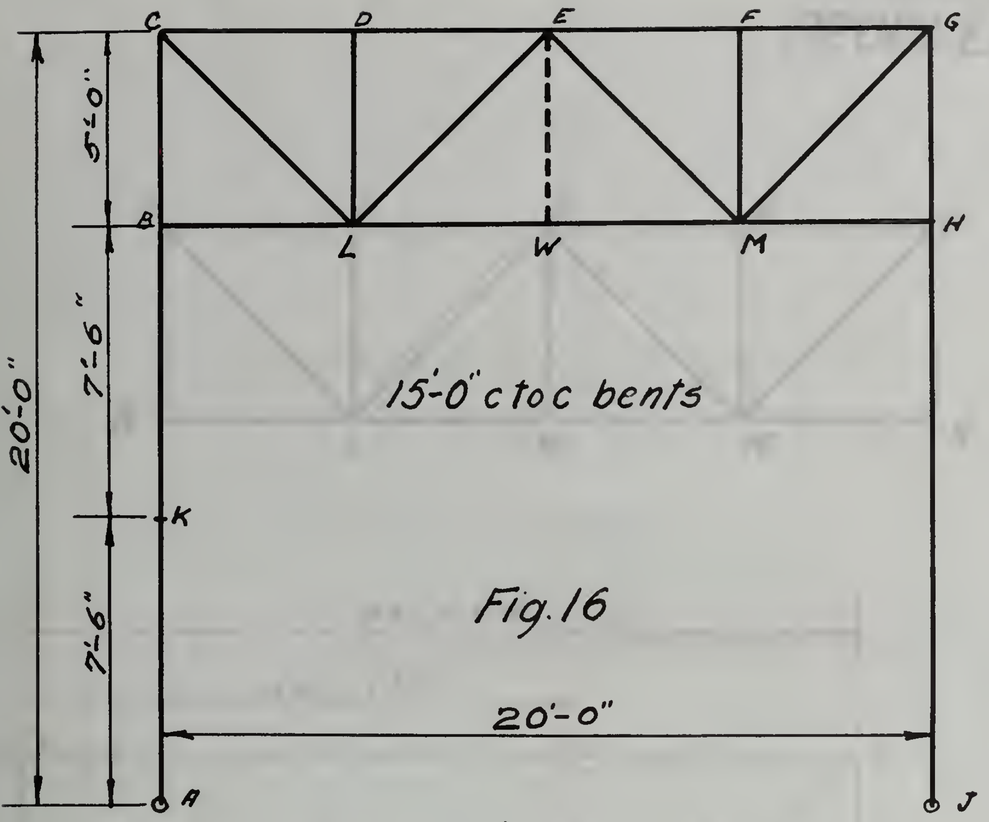

C
D
E
F
G
B
L
W
M
H
K
A
J
5'-0"
7'-6"
7'-6"
20'-0"
20'-0"
15'-0" c to c bents

Fig. 16

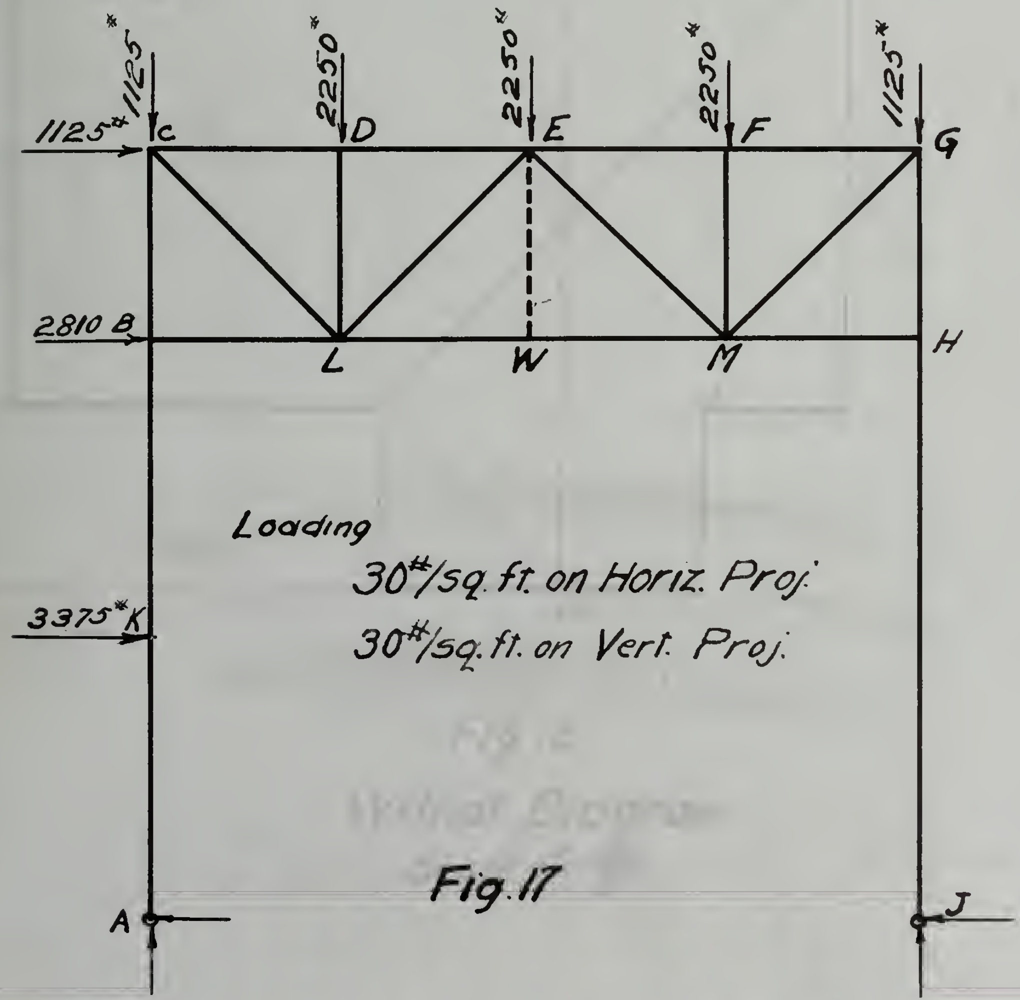
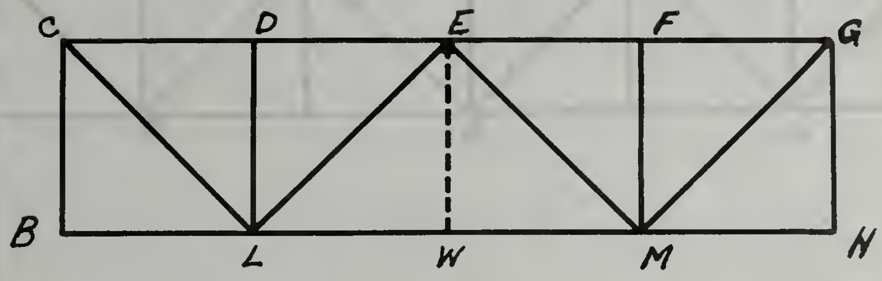

1125#
2250#
2250#
2250#
1125#
1125#
C
D
E
F
G
2810 B
L
W
M
H
3375# K
A
J
Loading
30#/sq. ft. on Horiz. Proj.
30#/sq. ft. on Vert. Proj.

Fig. 17

PROBLEM 4



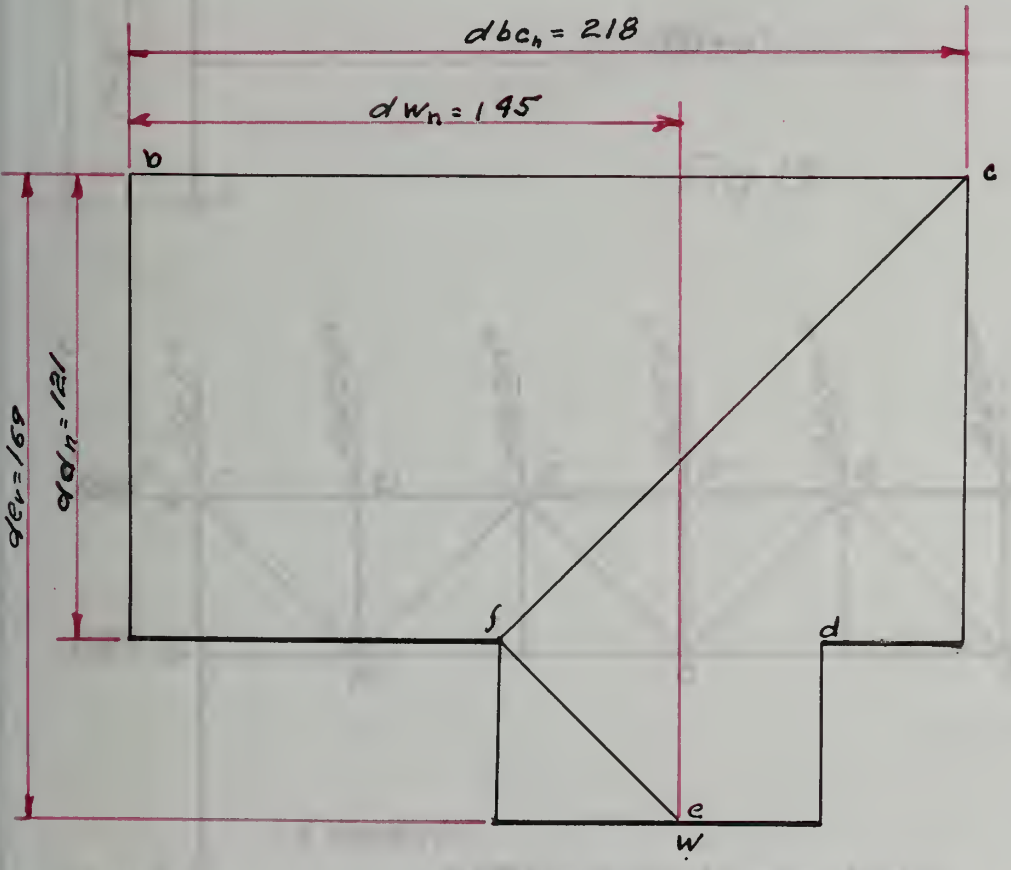


Fig. 18

Williot Diagram

Scale $1'' = \frac{50}{E}$

PROBLEM 4

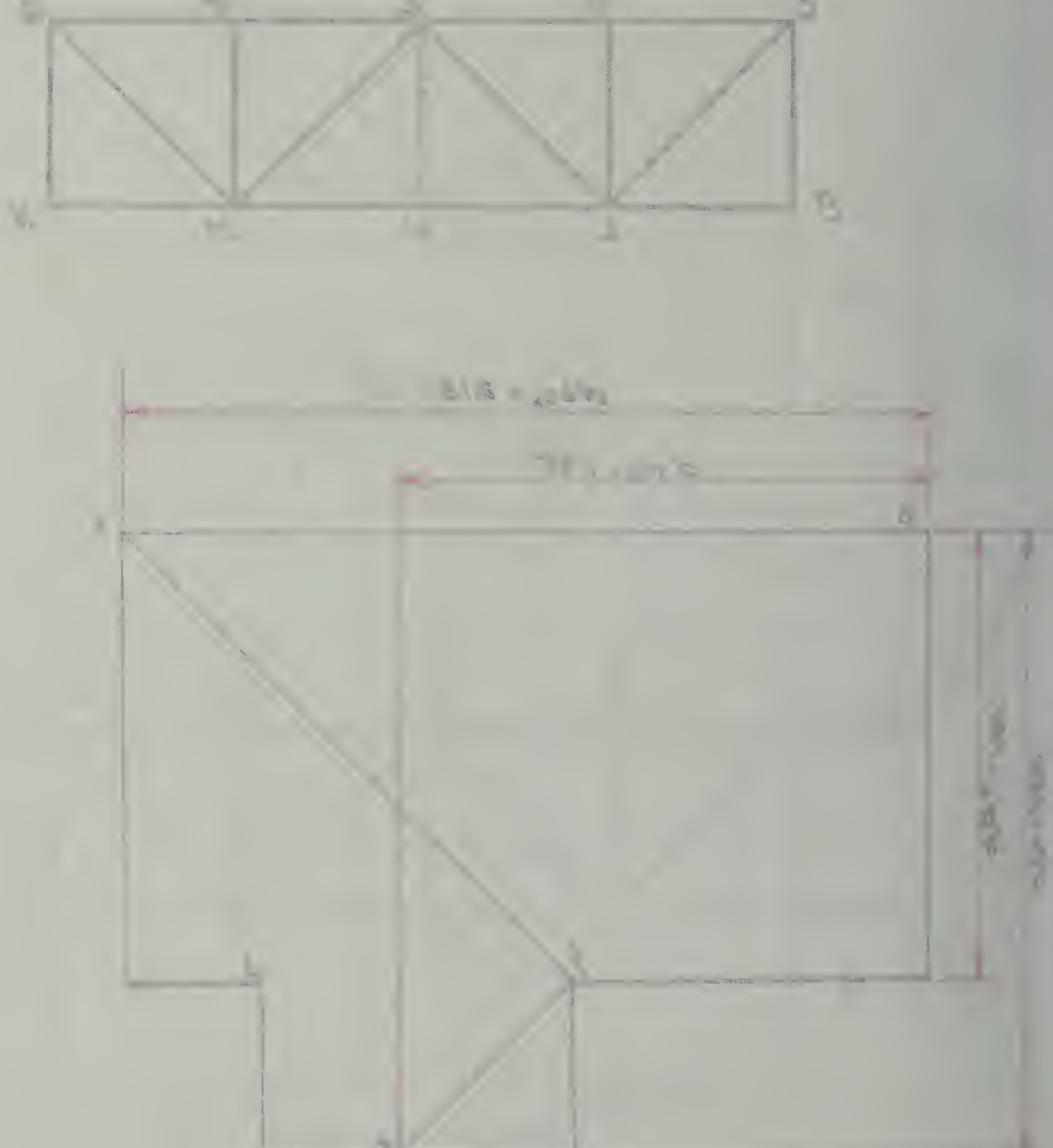


Fig 18

Williot Diagram

Scale 1" =

PROBLEM 5

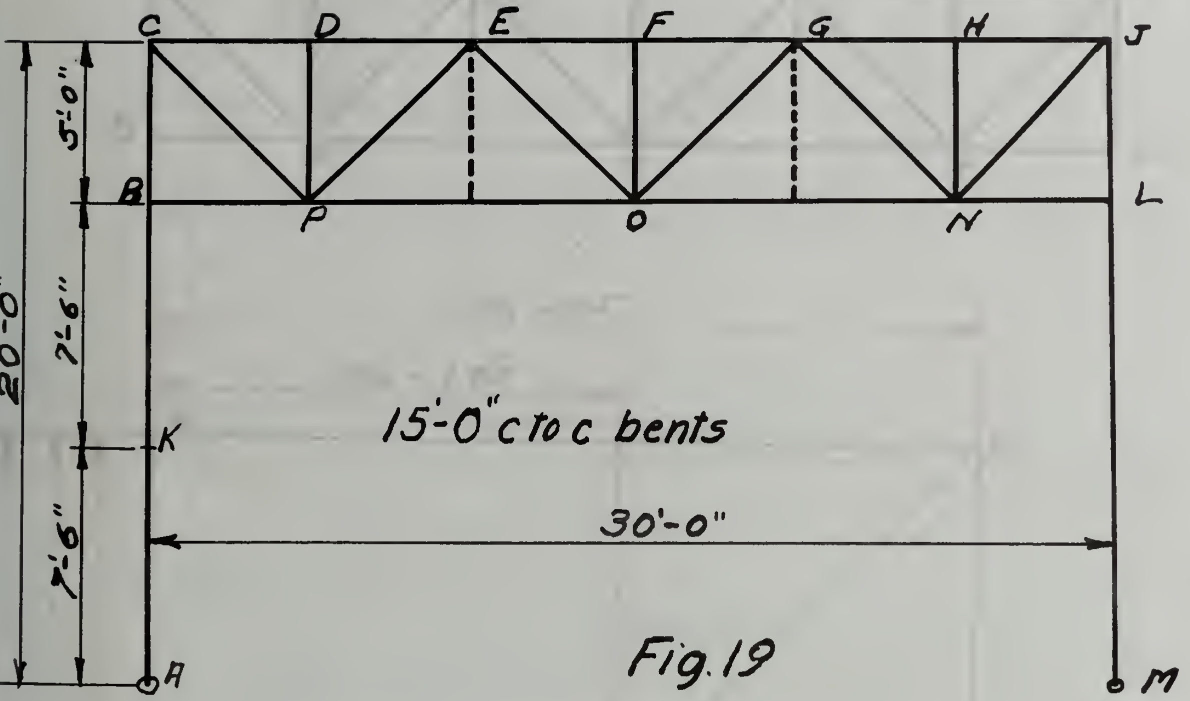


Fig. 19

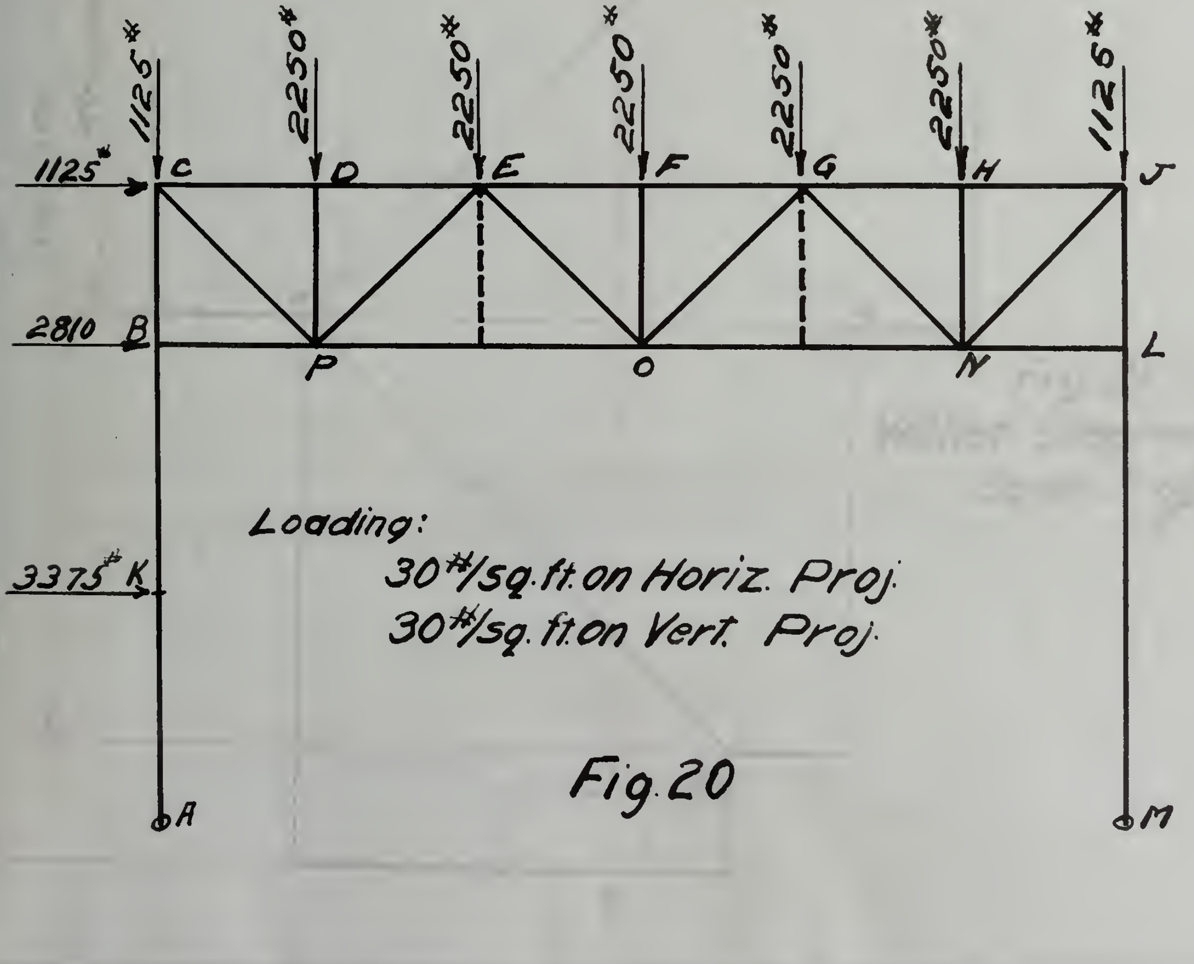
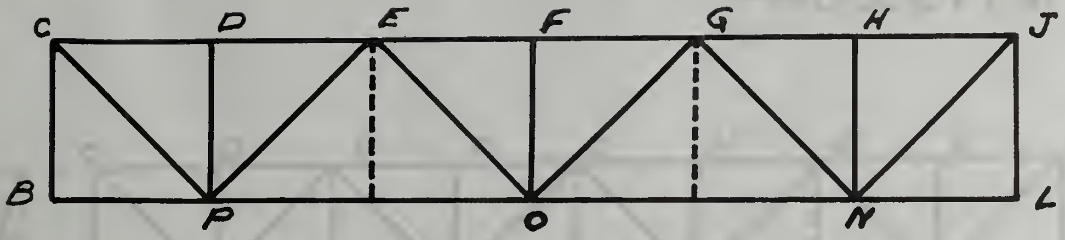


Fig. 20

PROBLEM 5



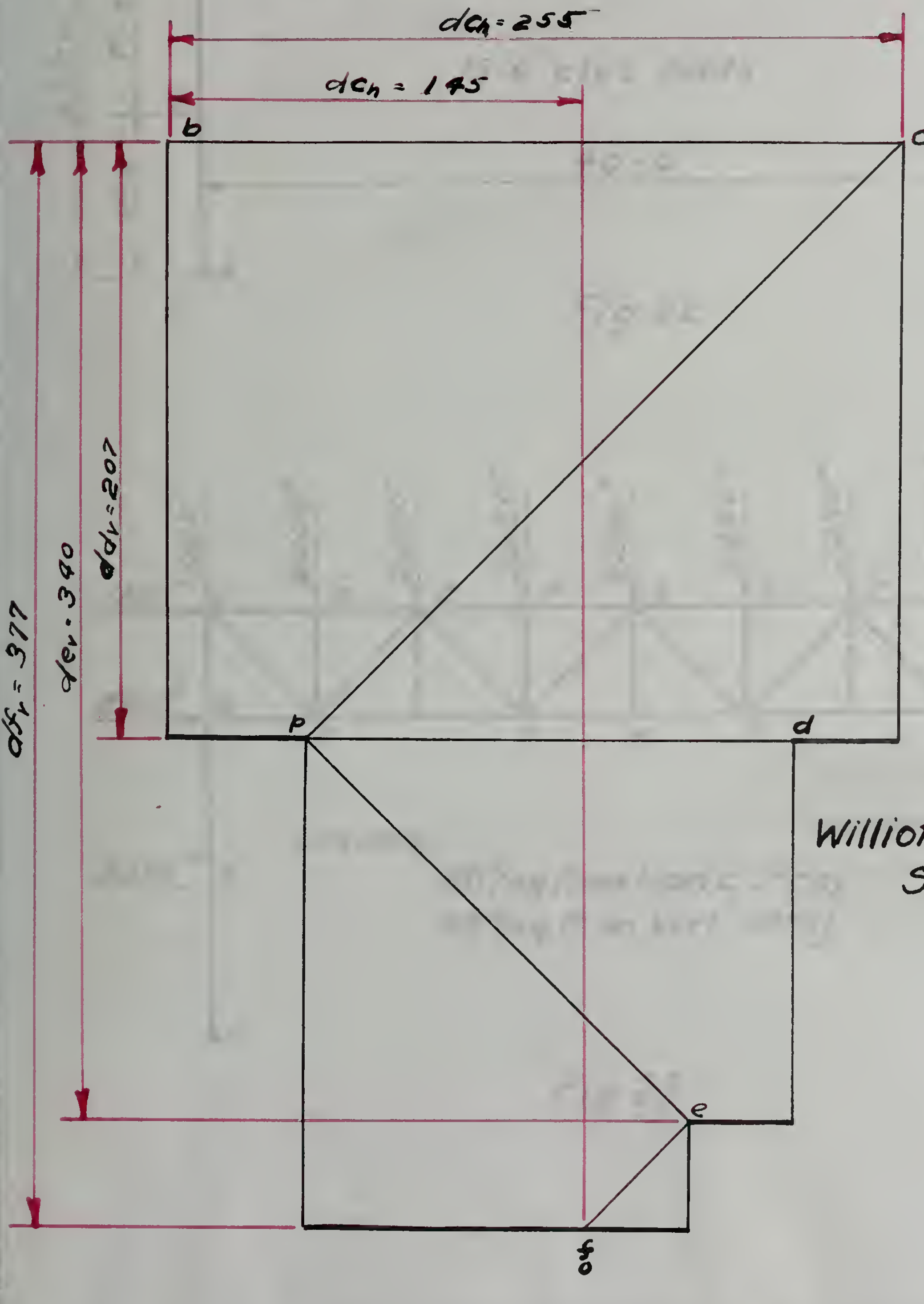


Fig. 21
Williot Diagram
Scale 1" = 60/E

PROBLEM 5

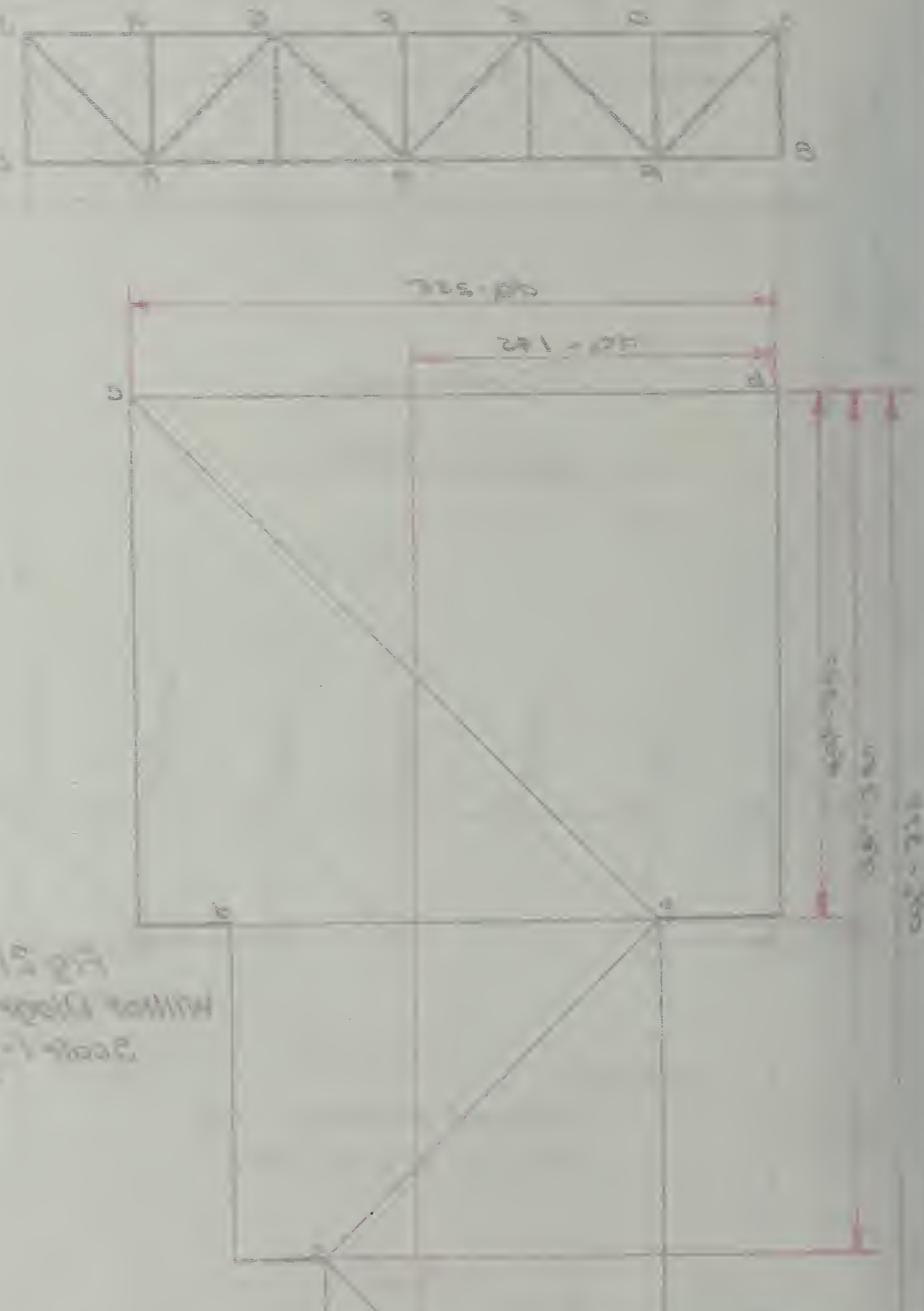


Fig 2
Maxwell Diagram

PROBLEM 6

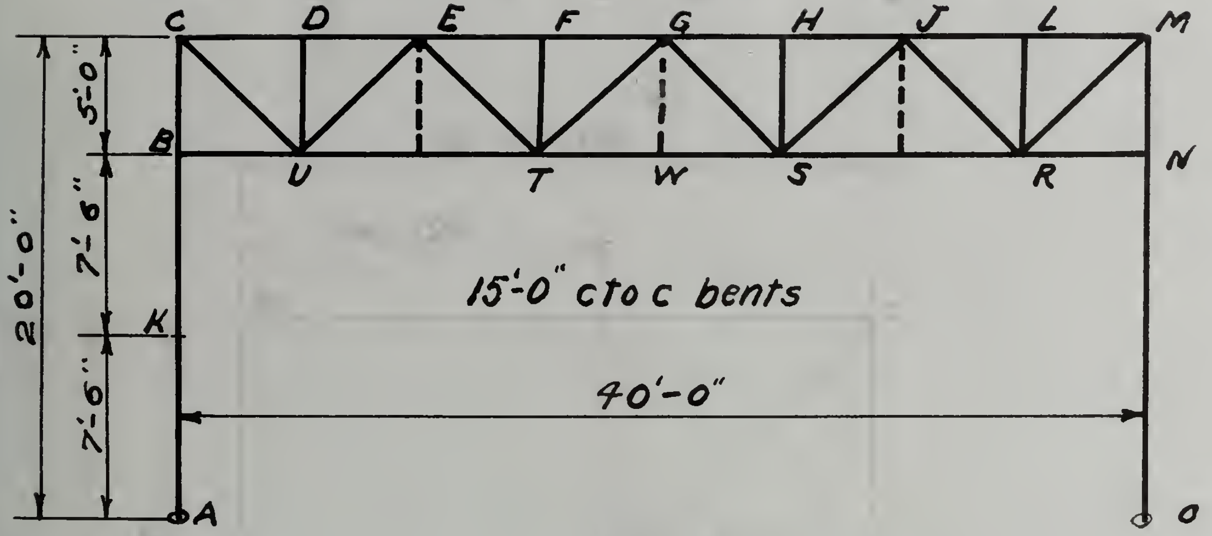


Fig. 22

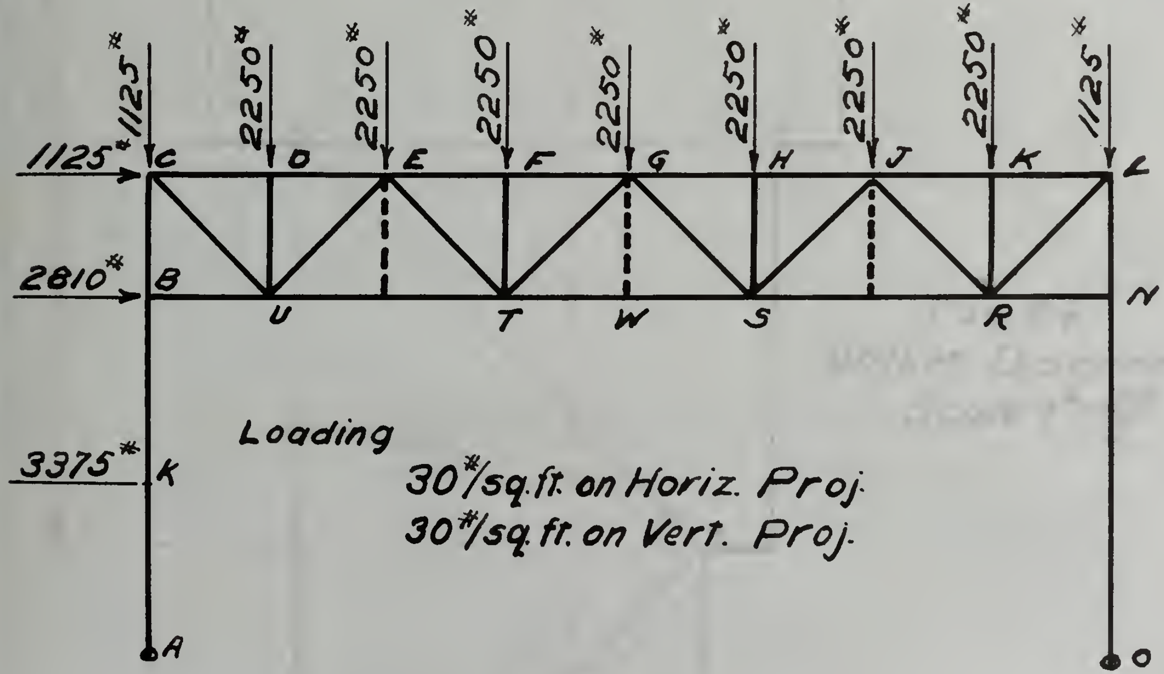
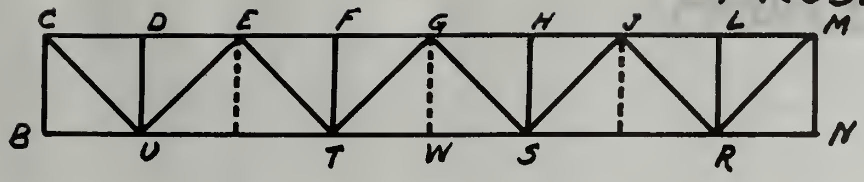


Fig. 23

PROBLEM 6



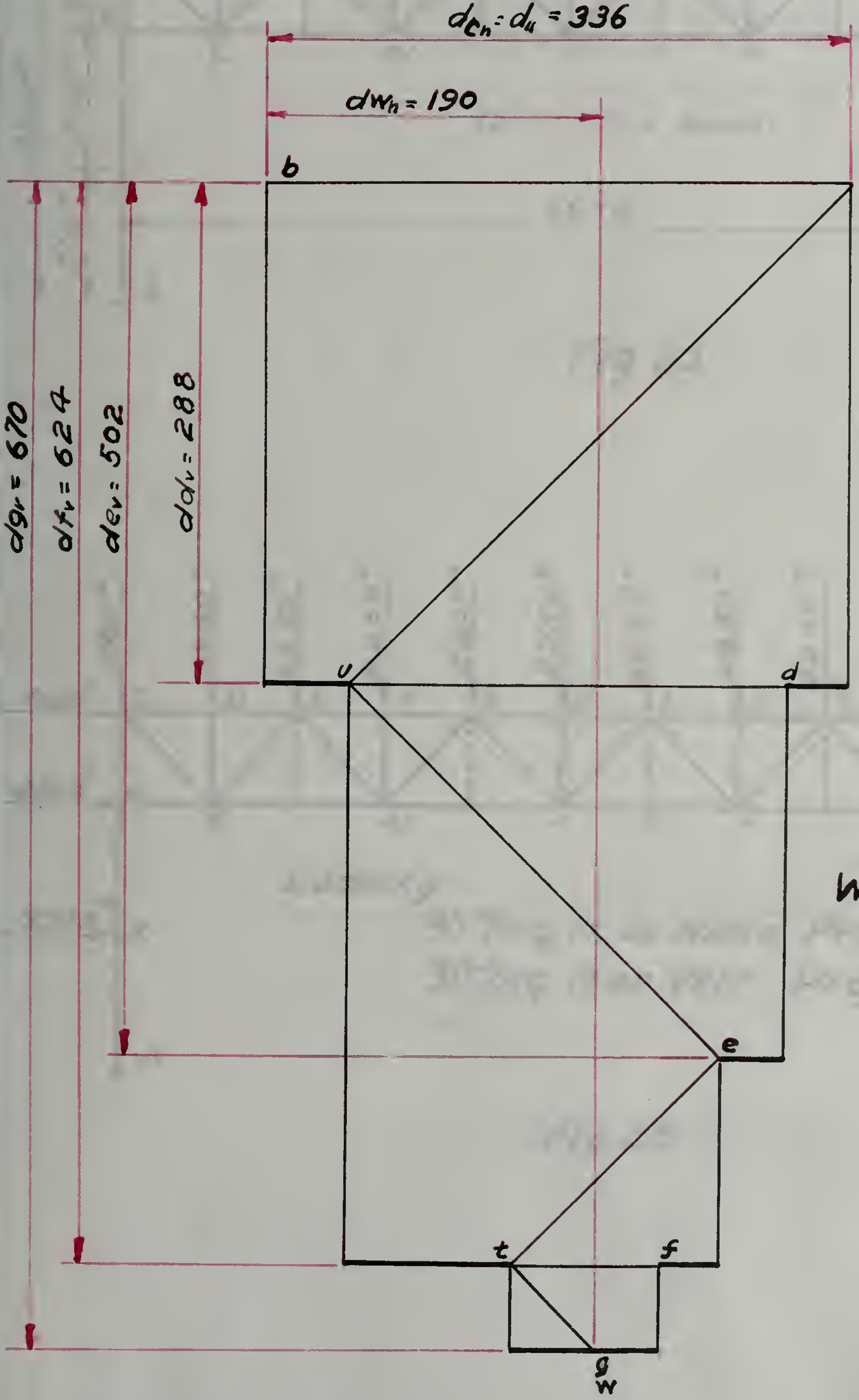


Fig. 24
Williot Diagram
Scale 1" = $\frac{100}{E}$

PROBLEM 7

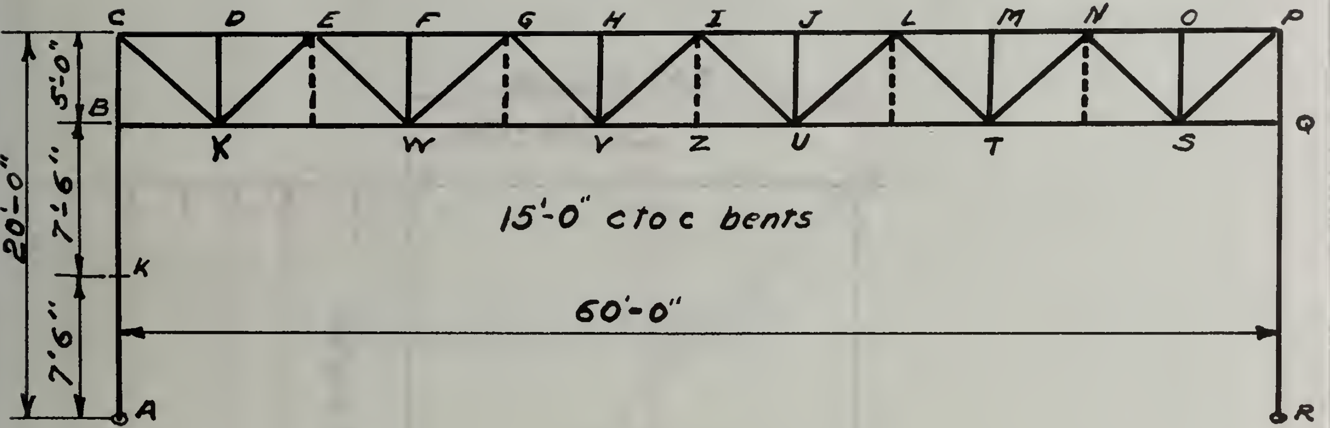


Fig. 25

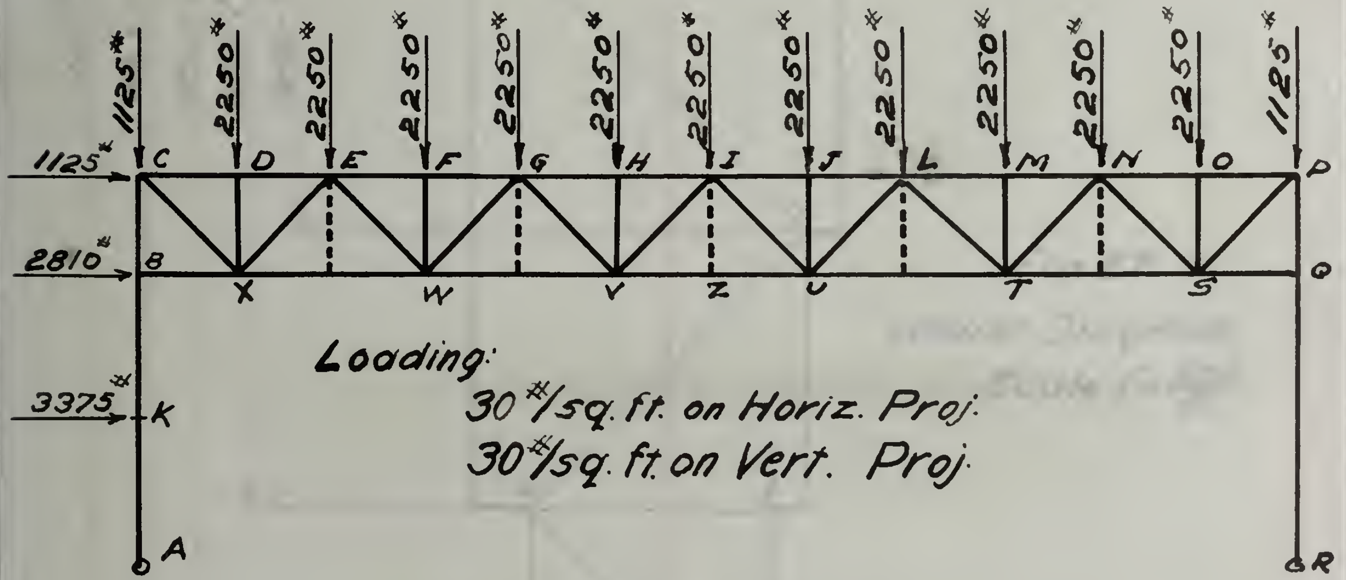
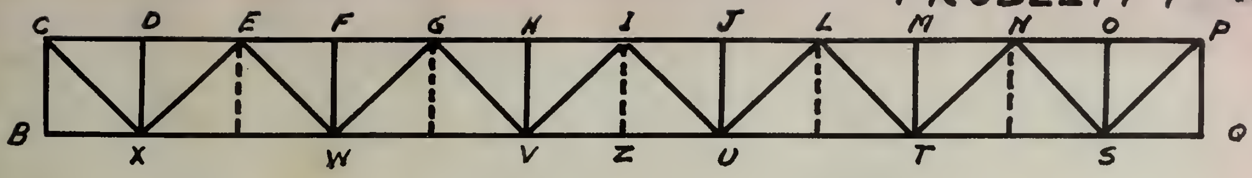


Fig. 26



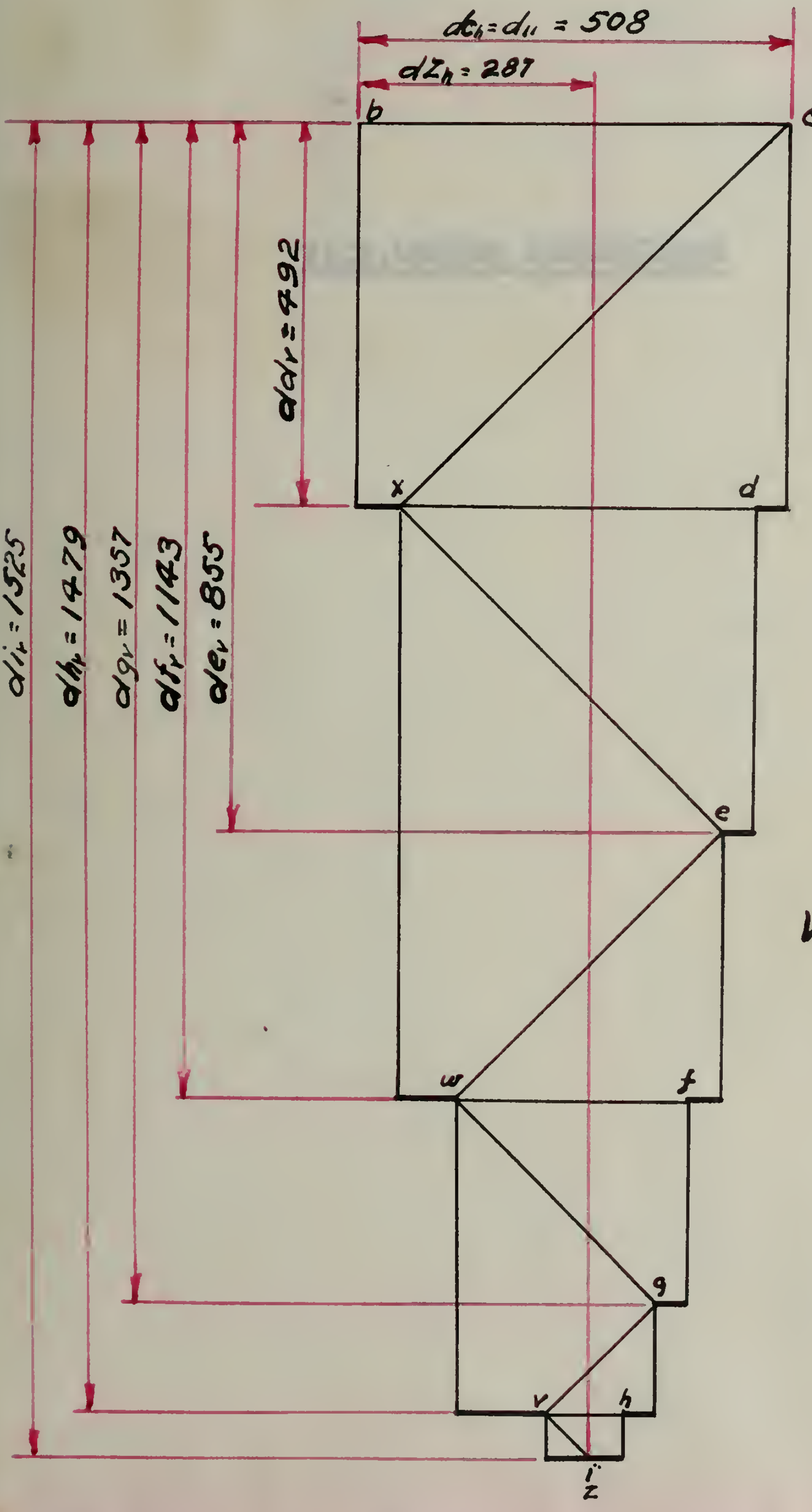


Fig. 27
Williot Diagram
Scale 1" = $\frac{200}{E}$

VIII. COLUMN DEFLECTIONS

[illegible]

PROBLEM 2.

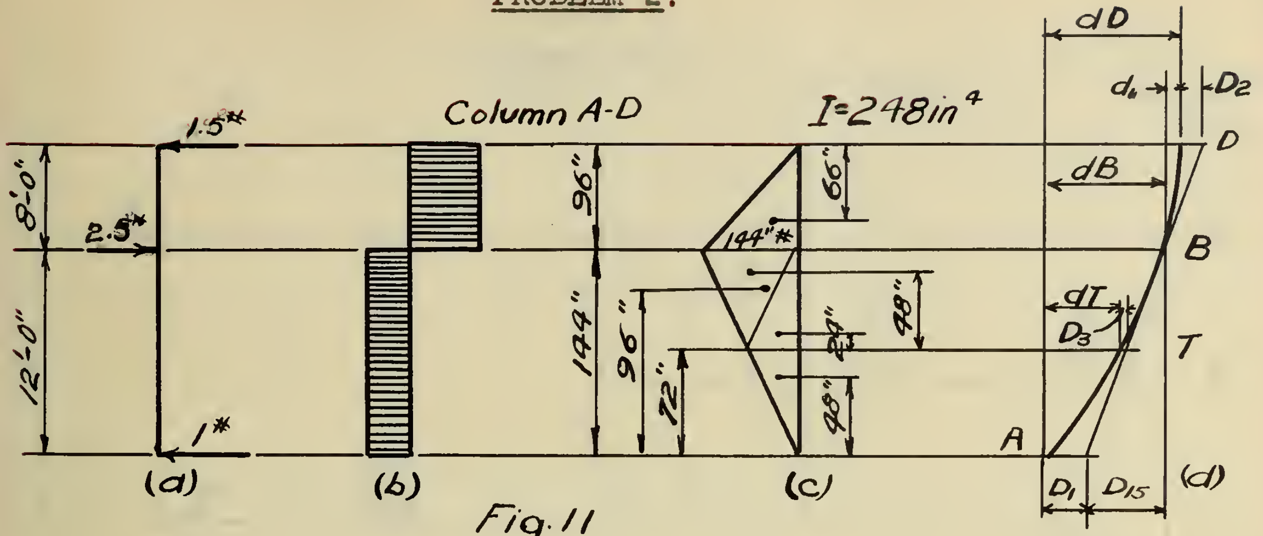


Fig. 11

$$D_2 = \frac{1}{E\,I}\left(\frac{144 \times 96 \times 64}{2}\right) = 1780 / E$$

$$D_1 = \frac{1}{E\,I}\left(\frac{144 \times 144 \times 96}{2}\right) = 4010 / E$$

$$D_3 = \frac{1}{E\,I}\left(\frac{144 \times 72 \times 48}{2} + \frac{72 \times 72 \times 24}{2}\right) = 1250 / E.$$

From the Williot diagram, $d_{11} = 770 / E$.

$D_{15} = 1.5\,(d_{11} + D_2) = 1.5(770 + 1780) = 3825 / E.$

$dB = D_{15} + D_1 = 7835 / E.$

$dD = dB + d_{11} = 8605 / E.$

$dT = dB - (\frac{1}{2}\, d_{15} + D_3) = 4673 / E.$

Total displacement of S from its original position $= 2\, dB +$

$1380 = 17050 / E.$

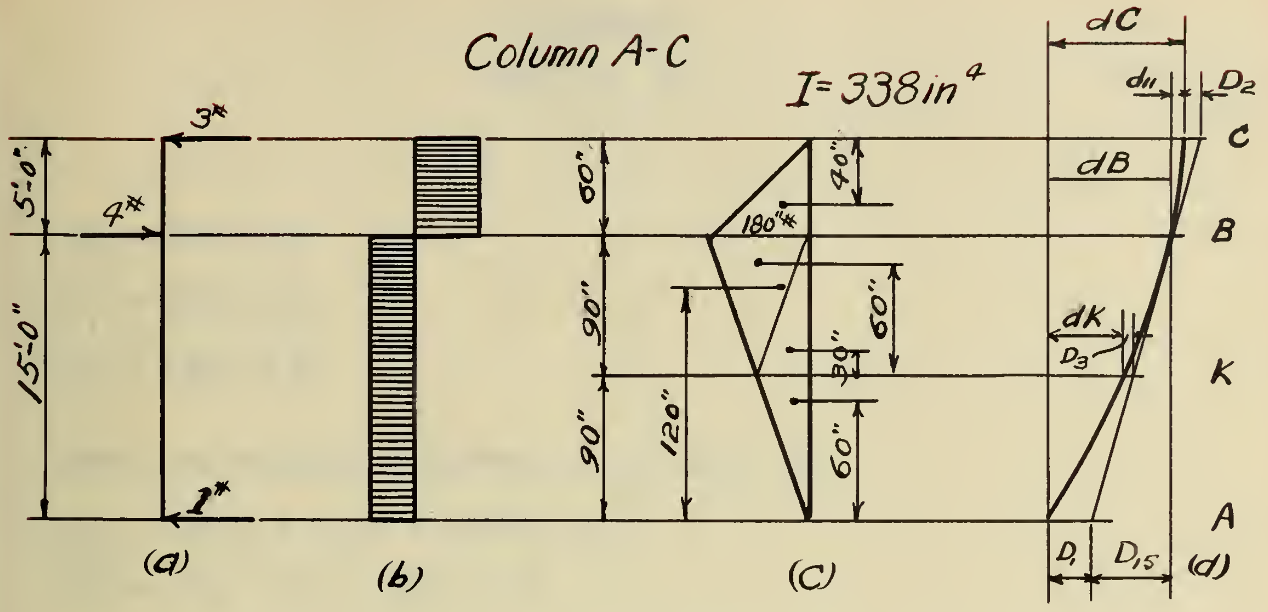


Fig. 15

PROBLEM 3.

See Fig. 15.

$$D_2 = \frac{1}{E`I}(80\times60\times40) = 640\ /\ E.$$

$$D_1 = \frac{1}{E`I}(90\times180\times120) = 5750\ /\ E.$$

$$D_3 = \frac{1}{E`I}\ (90\times90\times60\ +45\times90\times30) = 1800\ /E.$$

From the Williot diagram, $d_{11} = 85\ /\ E$.

$D_{15} = 3(D_2 + d_{11}) = 2175\ /\ E.$

$dB = D_{15} + D_1 = 7925\ /\ E.$

$dC = dB + d_{11} = 8010\ /\ E.$

$dK = dB - \frac{1}{2}\,D_{15} - D_3 = 5038\ /\ E.$

Total displacement of G from its original position

$= 2\ dB + 97 = 15947\ /\ E.$

PROBLEM 4.

See Fig. 15.

$D_2 = 640 / E.$

$D_1 = 5750 / E.$

$D_3 = 1800 / E.$

From the Williot diagram, $d_{11} = 216 / E.$

$D_{15} = 3(D_2 + d_{11}) = 2568 / E.$

$dB = D_{15} + D_1 = 8318 / E.$

$dC = dB + d_{11} = 8534 / E.$

$dK = dB - \frac{1}{2} D_{15} - D_3 = 5234 / E.$

Total displacement of J from its original position

$$= 2\ dB + 288 = 16924 / E.$$

PROBLEM 5.

$D_2 = 640 \ / \ E$

$D_1 = 5750 \ / \ E$

$D_3 = 1800 \ / \ E$

From the Williot diagram, $d_{11} = 255 \ / \ E$.

$D_{15} = 3(D_2 + d_{11}) = 2685 \ / \ E.$

$dB = D_{15} + D_1 = 8435 \ / \ E.$

$dC = dB + d_{11} = 8690 \ / \ E.$

$dK = dB - \frac{1}{2} D_{15} - D_3 = 5293 \ / \ E.$

Total displacement of M from its original position

$= 2 \ dB + 290 = 17160 \ / \ E.$

PROBLEM 6.

See Fig. 15.

$D_2 = 640 / E.$

$D_1 = 5750 / E.$

$D_3 = 1800 / E.$

From the Williot diagram, $d_{11} = 336 / E.$

$D_{15} = 3(D_2 + d_{11}) = 2928 / E.$

$dB = D_{15} + D_1 = 8678 / E.$

$dC = dB + d_{11} = 9014 / E.$

$dK = dB - \frac{1}{2} D_{15} - D_3 = 5414 / E.$

Total displacement of O from its original position

$$= 2\ dB + 380 = 17736 / E.$$

PROBLEM 7.

See Fig. 15.

$D_2 = 640 / E.$

$D_1 = 5750 / E.$

$D_3 = 1800 / E.$

From the Williot diagram, $d_{11} = 508 / E.$

$D_{15} = 3(D_2 + d_{11}) = 3444 / E.$

$dB = D_{15} + D_1 = 9194 / E.$

$dC = dB + d_{11} = 9702 / E.$

$dK = dB - \frac{1}{2} D_{15} - D_3 = 5672 / E.$

Total displacement of **R** from its original position

$$= 2\ dB + 574 = 18962 / E.$$

IX. TABLES AND CURVES

TABLE 1.

1	2	3	4	5	6	7
Member	Length L inches	Material	Area A sq. in.	L/A	Stress U lb./sq.in	Strain U×L/A in./E
A B	60	1-Pl- 18 × 3/8" 4-Ls- 5×3×7/16"	19.99	3.0	- 3.77	- 11.3
B C	93	2-Ls-3×2 1/2×1/4"	2.62	34.1	- 8.41	-287.0
C D	93	do.	2.62	34.13	- 4.23	- 144.3
D E	93	do.	2.62	34.13	- 2.82	- 96.2
E F	93	do.	2.62	34.13	- 2.82	- 96.2
F G	93	do.	2.62	34.13	- 4.23	-144.3
G H	93	do.	2.62	34.13	- 8.41	-287.0
H J	60	1-Pl- 18 × 3/8" 4-Ls- 5×3×7/16"	19.99	3.0	- 3.77	- 11.3
J O	100	2-Ls- 5×3×5/16"	4.80	20.82	+ 6.26	+ 13.0
O N	80	2-Ls- 2 1/2×2×1/4"	2.12	37.72	+ 8.52	+321.5
N M	80	do.	2.12	37.72	+ 4.77	+180.0
M L	80	do.	2.12	37.72	+ 4.77	+ 180.0
L K	80	do.	2.12	37.72	+ 8.52	+ 321.5
K A	100	2-Ls-5×3×5/16"	4.80	20.82	+ 6.26	+ 13.0
B K	80	2-Ls-2 1/2×2×1/4"	2.12	37.72	+ 3.52	+ 133.0
C K	40	2-Ls-2 1/2×2×5/16"	2.62	15.25	+ 3.77	+ 57.6
C L	93	2-Ls-2 1/2×2×1/4"	2.12	42.20	- 4.23	- 178.5
D L	80	do.	2.12	37.72	+ 1.89	+ 71.3
D M	113	do.	2.12	53.35	- 1.78	- 94.9
E M	120	do.	2.12	56.60	+ 2.53	+ 143.2
M F	113	do.	2.12	53.35	- 1.78	- 94.9
F N	80	do.	2.12	37.72	+ 1.89	+ 71.3
N G	93	do.	2.12	42.20	- 4.23	- 178.5
G O	40	2-Ls-2 1/2×2×5/16"	3.62	15.25	+ 3.77	+ 57.6
O H	80	2-Ls-2 1/2×2×1/4"	2.12	37.72	133.0	

TABLE 2.

	1	2	3	4	5
Direction of Load	Point	Load lb.	Deflection of Point due to 1 lb. at P in./E	2 × 3	$\frac{2 \times 3}{27360/E}$ lb.
Horizontal	R	4500	8990	40450000	1480
"	A	3375	11480	38750000	1415
"	B	1125	13100	14750000	539
Normal	B	1250	730	912000	33
"	C	2500	3050	7620000	278
"	D	2500	3280	8200000	300
"	E	1250	3100	3875000	142
Vertical	B	1500	11	16500	1
"	C	3000	2220	6660000	243
"	D	3000	2490	7470000	273
"	E	3000	2360	7080000	259
"	F	3000	2490	7470000	273
"	G	3000	2220	6660000	243
"	H	1500	11	16500	1

Horizontal component of reaction at P = 5480 lb.

[illegible]

TABLE 3

1	2	3	4	5	6	7
Member	Length L inches	Material	Area A sq. in.	L/A	Stress U lb./sq.in	Strain U×L/A in./E
B D	96	1-Pl- 10 × 3/8" 4-Ls-5×3×3/8"	15.19	6.34	- 1.33	- 8.4
D F	182	2-Ls-3×3×3/8"	4.22	43.20	- 3.22	-139.0
F H	182	do.	4.22	43.20	- 2.31	- 99.8
H J	182	do.	4.22	43.20	- 3.31	- 99.8
J L	182	do.	4.22	43.20	- 3.22	-139.0
L N	96	1-Pl- 10 × 3/8" 4-Ls- 5×3×3/8"	15.19	6.34	- 1.33	- 8.4
N O	102	2-Ls-3×3×3/8"	4.22	24.20	+ 2.83	+ 58.4
O P	180	do.	4.22	42.70	+ 3.67	+156.7
P Q	180	do.	4.22	42.70	+ 3.01	+128.5
Q R	180	do.	4.22	42.70	+ 3.67	+156.7
R B	102	do.	4.22	24.20	+ 2.83	+ 58.4
D R	102	1-L-3×3×3/8"	2.11	48.40	+ 1.93	+ 93.4
R F	115	2-Ls-2 1/2×2×5/16"	2.62	44.00	+ 0.57	+ 25.1
F Q	115	1-L- 2 1/2×2×5/16"	1.31	88.00	- 0.48	- 42.2
Q H	132	do.	1.31	100.50	+ 0.42	+ 42.2
H P	132	do.	1.31	100.50	+ 0.42	+ 42.2
P J	115	do.	1.31	88.00	- 0.48	- 42.2
J O	115	2-Ls-2 1/2×2×5/16"	2.62	44.00	+ 0.57	+ 25.1
O L	102	1-L- 3×3×3/8"	2.11	48.40	+ 1.93	+ 93.4

Other members have no stress and consequently do not enter the problem.

TABLE 4.

	1	2	3	4	5
Direction of Load	Point	Load lb.	Deflection of Point due to 1 lb. at P	2 × 3	$\frac{2 \times 3}{17050}$
Horizontal	T	2700	4673	12620000	740
"	B	3150	7835	24650000	1445
"	D	1800	8605	15500000	910
Vertical	D	1687	8	13500	1
"	E	3375	780	2630000	154
"	F	3375	1125	3790000	223
"	G	3375	1245	4200000	246
"	H	3375	1245	4200000	246
"	I	3375	1245	4200000	246
"	J	3375	1125	3790000	223
"	K	3375	780	2630000	154
"	L	1687	8	13500	1

Horizontal component of reaction at P = 4589#

[illegible]

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TABLE 5

1	2	3	4	5	6	7
Member	Length L inches	Material	Area A sq. in.	L/A	Stress U lb./sq.in	Strain U×L/A in./E
C D	60	2-Ls-3 1/2×3 1/2×3/8"	4.96	12.10	- 3.0	- 36.20
D E	60	do.	4.96	12.10	- 3.0	- 36.20
F H	60	do.	4.96	12.10	+ 4.0	+ 48.40
H B	60	do.	4.96	12.10	+ 4.0	+ 49.40

Other members have no stress and hence do not enter the problem.

TABLE 6.

	1	2	3	4	5
Direction of Load	Point	Load lb.	Deflection of Point	2 × 3	$\frac{2 \times 3}{15947}$
Horizontal	K	3375	5038	17000000	1065
"	B	2810	7925	22300000	1400
"	C	1125	8010	9010000	565
Vertical	C	1125	0	0	0
"	D	2250	36	81000	5
"	E	1125	0	0	0
					3035 lb.

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TABLE 7.

1	2	3	4	5	6	7
Member	Length L inches	Material	Area A sq. in.	L/A	Stress U lb./sq.in	Strain U×L/A in./E
C E	120	2-Ls-3 1/2×3 1/2×3/8"	4.96	24.2	- 3.0	- 72.6
E G	120	do.	4.96	24.2	- 3.0	- 72.6
H M	60	do.	4.96	12.1	+ 4.0	+ 48.4
M L	120	do.	4.96	24.2	+ 4.0	+ 96.8
L B	60	do.	4.96	12.1	+ 4.0	+ 48.4
		Other members have no stress.				

TABLE 8.

	1	2	3	4	5
Direction of Load	Point	Load lb.	Deflection of Point	2 × 3	$\frac{2 \times 3}{16924}$
Horizontal	K	3375	5234	17700000	1045
"	B	2810	8318	23300000	1380
"	C	1125	8534	9580000	567
Vertical	C	1125	0	0	0
"	D	2250	120	270000	16
"	E	2250	170	382000	23
"	F	2250	120	270000	16
"	G	1125	0	0	0
					3047 lb.

TABLE 9.

1	2	3	4	5	6	7
Member	Length L inches	Material	Area A sq. in.	L/A	Stress U lb./sq.in	Strain U×L/A in./E
C E	120	2-Ls-3 1/2×3 1/2×3/8"	4.96	24.2	- 3.0	- 72.6
E G	120	do.	4.96	24.2	- 3.0	- 72.6
G J	120	do.	4.96	24.2	- 3.0	- 72.6
L N	60	do.	4.96	12.1	+ 4.0	+ 48.4
N O	120	do.	4.96	24.2	+ 4.0	+ 96.8
O P	120	do.	4.96	24.2	+ 4.0	+ 96.8
P B	60	do.	4.96	12.1	+ 4.0	+ 48.4

Other members have no stress.

TABLE 10.

	1	2	3	4	5
Direction	Point	Load	Deflection	2 × 3	$\frac{2 \times 3}{17160}$
Horizontal	K	3375	5293	17900000	1045
"	B	2810	8435	23700000	1390
"	C	1125	8690	9780000	573
Vertical	C	1125	0	0	0
"	D	2250	207	466000	27
"	E	2250	340	765000	45
"	F	2250	377	850000	50
"	G	2250	340	765000	45
"	H	2250	207	466000	27
"	J	1125	0	0	0
					3202 lb.

[illegible]

[illegible]

TABLE 11.

1	2	3	4	5	6	7
Member	Length L inches	Material	Area A sq. in.	L/A	Stress U lb./sq.in	Strain U×L/A in./E
C E	120	2-Ls-3 1/2×3 1/2×3/8"	4.96	24.2	- 3.0	-72.6
E G	120	do.	4.96	24.2	- 3.0	-72.6
G J	120	do.	4.96	24.2	- 3.0	-72.6
J M	120	do.	4.96	24.2	- 3.0	-72.6
N R	60	do.	4.96	12.1	+ 4.0	+48.4
R S	120	do.	4.96	24.2	+ 4.0	+96.8
S T	120	do.	4.96	24.2	+ 4.0	+96.8
T U	120	do.	4.96	24.2	+ 4.0	+96.8
U B	60	do.	4.96	24.2	+ 4.0	+96.8

TABLE 12.

	1	2	3	4	5
Direction	Point	Load	Deflection	2 × 3	$\frac{2 \times 3}{17736}$
Horizontal	K	3375	5415	18250000	1030
"	B	2810	8678	24400000	1375
"	C	1125	9014	10150000	572
Vertical	C	2250	0	0	0
"	D	2250	288	650000	37
"	E	2250	502	1130000	64
"	F	2250	624	1400000	79
"	G	2250	670	1510000	85
"	H	2250	624	1400000	79
"	J	2250	502	1130000	64
"	L	2250	288	650000	37
"	M	1125	0	0	0
					3402 lb.

TABLE 13.

1	2	3	4	5	6	7
Member	Length L inches	Material	Area A sq. in.	L/A	Stress U lb./sq.in	Strain U×L/A in./E
C E	120	2-Ls-3 1/2×3 1/2× 3/8"	4.96	24.2	- 3.0	- 72.6
E G	120	do.	4.96	24.2	- 3.0	- 72.6
G I	120	do.	4.96	24.2	- 3.0	- 72.6
I L	120	do.	4.96	24.2	- 3.0	- 72.6
L N	120	do.	4.96	24.2	- 3.0	- 72.6
N P	120	do.	4.96	24.2	- 3.0	- 72.6
Q S	60	do..	4.96	12.1	+ 4.0	+ 48.4
S T	120	do.	4.96	24.2	+ 4.0	+ 96.8
T U	120	do.	4.96	24.2	+ 4.0	+ 96.8
U V	120	do.	4.96	24.2	+ 4.0	+ 96.8
V W	120	do.	4.96	24.2	+ 4.0	+ 96.8
W X	120	do.	4.96	24.2	+ 4.0	+ 96.8
X B	60	do.	4.96	12.1	+ 4.0	+ 48.4
	Other members have no stress.					

TABLE 14.

	1	2	3	4	5
Direction	Point	Load	Deflection	2 × 3	$\frac{2 \times 3}{18962}$
Horizontal	K	3375	5672	19150000	1010
"	B	2810	9194	25800000	1360
"	C	1125	9702	10900000	576
Vertical	C	1125	0	0	0
"	D	2250	462	1017000	55
"	E	2250	855	1920000	102
"	F	2250	1143	2570000	136
"	G	2250	1357	3050000	161
"	H	2250	1479	3320000	176
"	I	2250	1525	3430000	181
"	J	2250	1479	3320000	176
"	L	2250	1357	3050000	161
"	M	2250	1143	2570000	136
"	N	2250	855	1920000	102
"	O	2250	462	1017000	55
"	P	1125	0	0	0
					4387 lb.

TABLE 15.

1	2	3	4	5
Problem No.	Ratio Length to Height	Ratio of Total H component of Lee Reaction to Total H Loads	Ratio of H component due to H loads to H Loads (Lee Reaction)	Ratio of H component due to Vertical Loads to Vertical Loads (Lee Reaction)
1	1.14	44.4 %	30.5 %	7.95 %
2	2.50	60.0	40.5	5.53
3	0.50	41.5	41.4	0.11
4	1.00	41.7	40.9	0.61
5	1.50	43.8	41.0	1.44
6	2.00	46.6	40.4	2.48
7	3.00	60.0	40.3	5.34

TABLE 16.

PERCENTAGE OF SINGLE HORIZONTAL LOAD REACHING LEE REACTION

	Ratio	Point of Application of Load- per cent of col.ht.		
Prob. No.	Lgth./Ht.	37.5	75.0	100.0
3	0.5	31.50	49.75	50.30
4	1.0	30.90	49.10	50.30
5	1.5	30.95	49.30	50.80
6	2.0	30.50	48.90	50.80
7	3.0	30.0	48.50	51.10

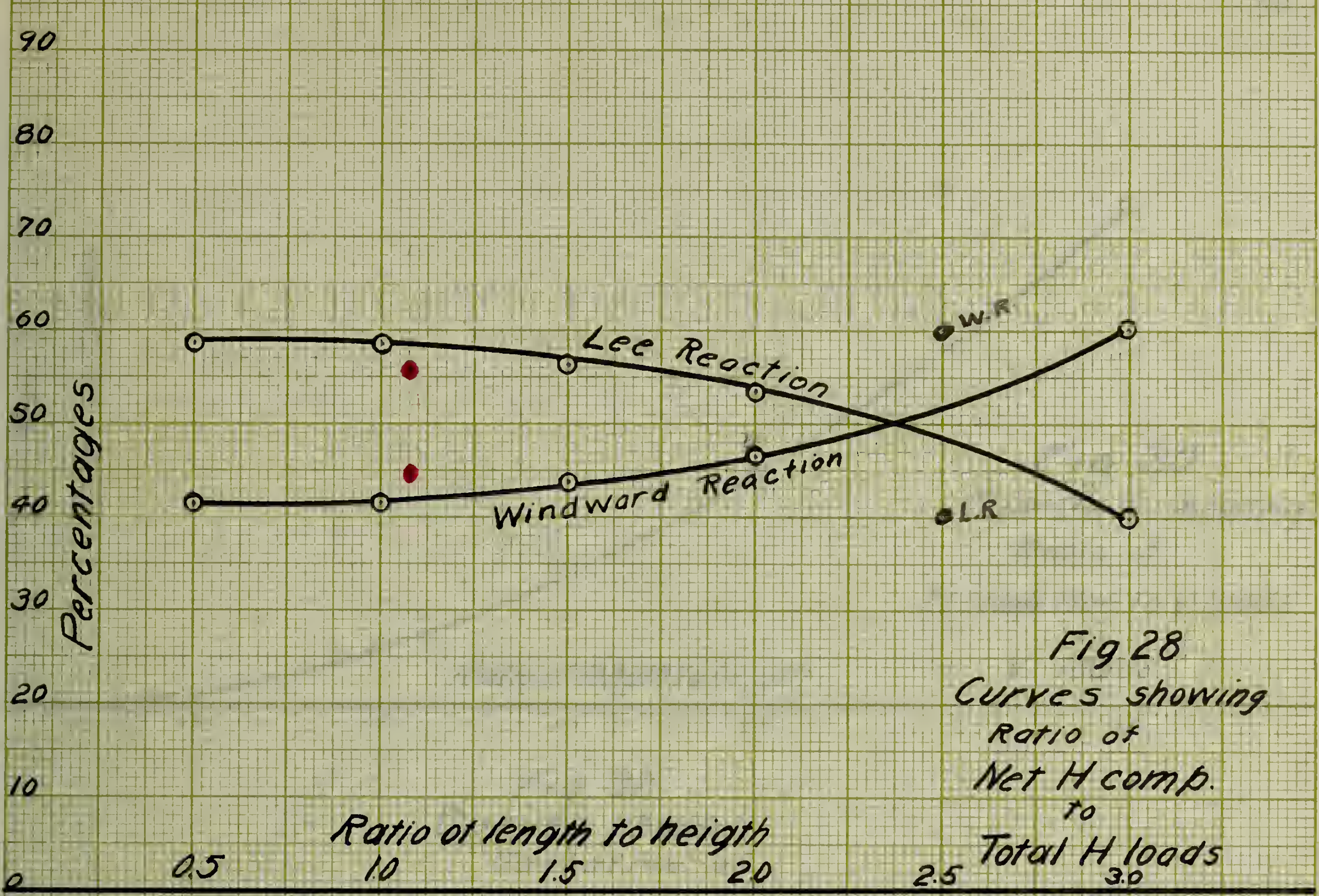


Fig 28
Curves showing
Ratio of
Net H comp.
to
Total H loads

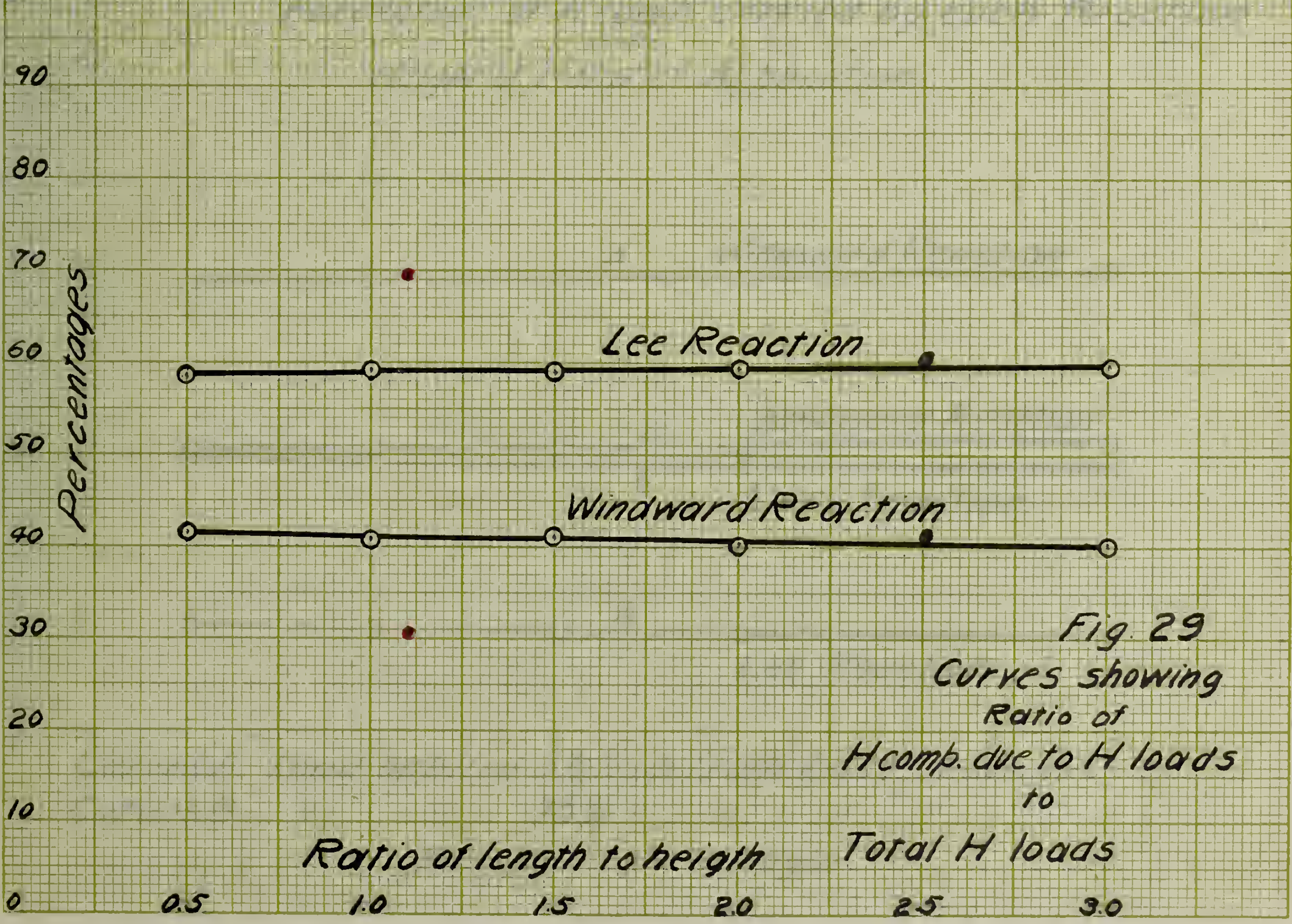
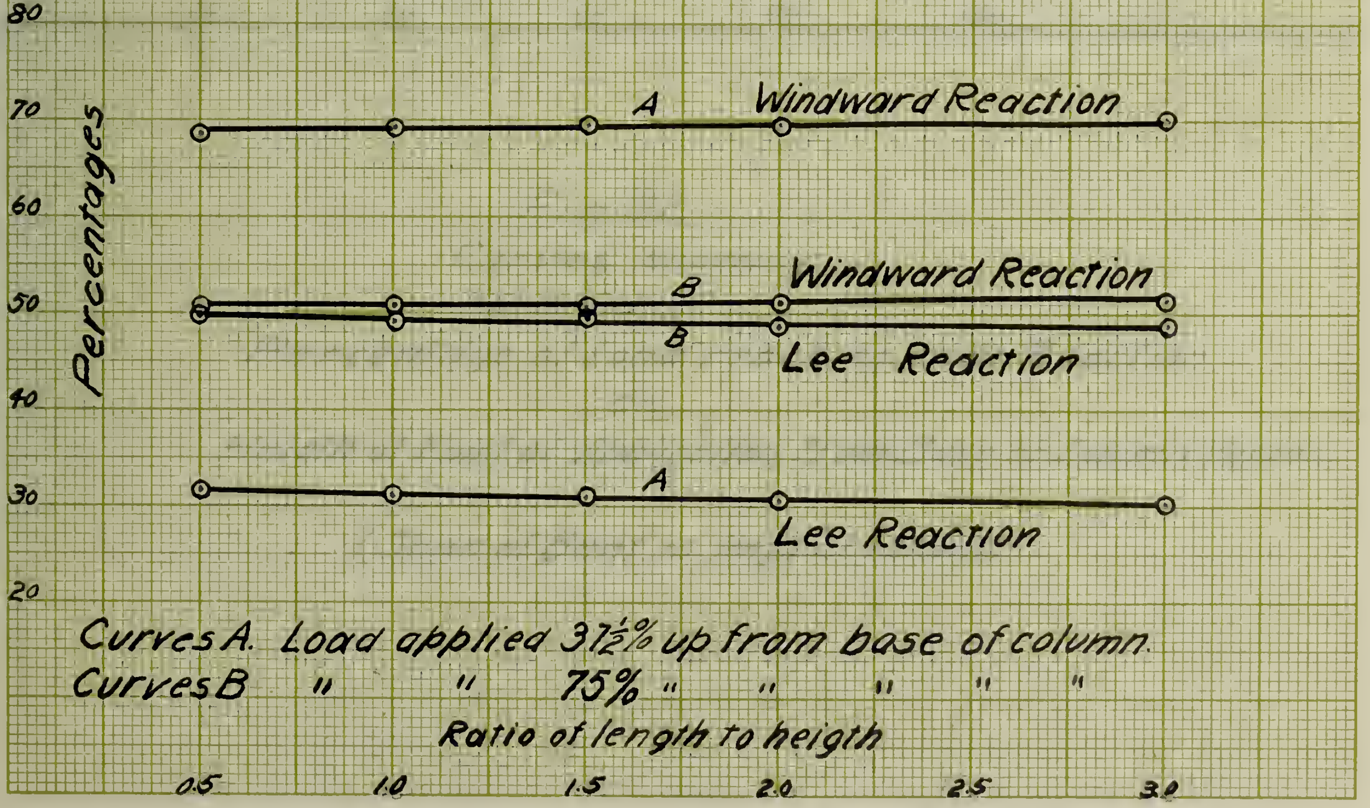


Fig. 29
Curves showing
Ratio of
H comp. due to H loads
to
Total H loads

Fig. 30
Curves showing
Ratio of
H. comp due to V loads
to
V loads

Fig. 32
Curves showing
Variation of
Percentage of H comp. carried by each Reaction
with
Height of Point of Application



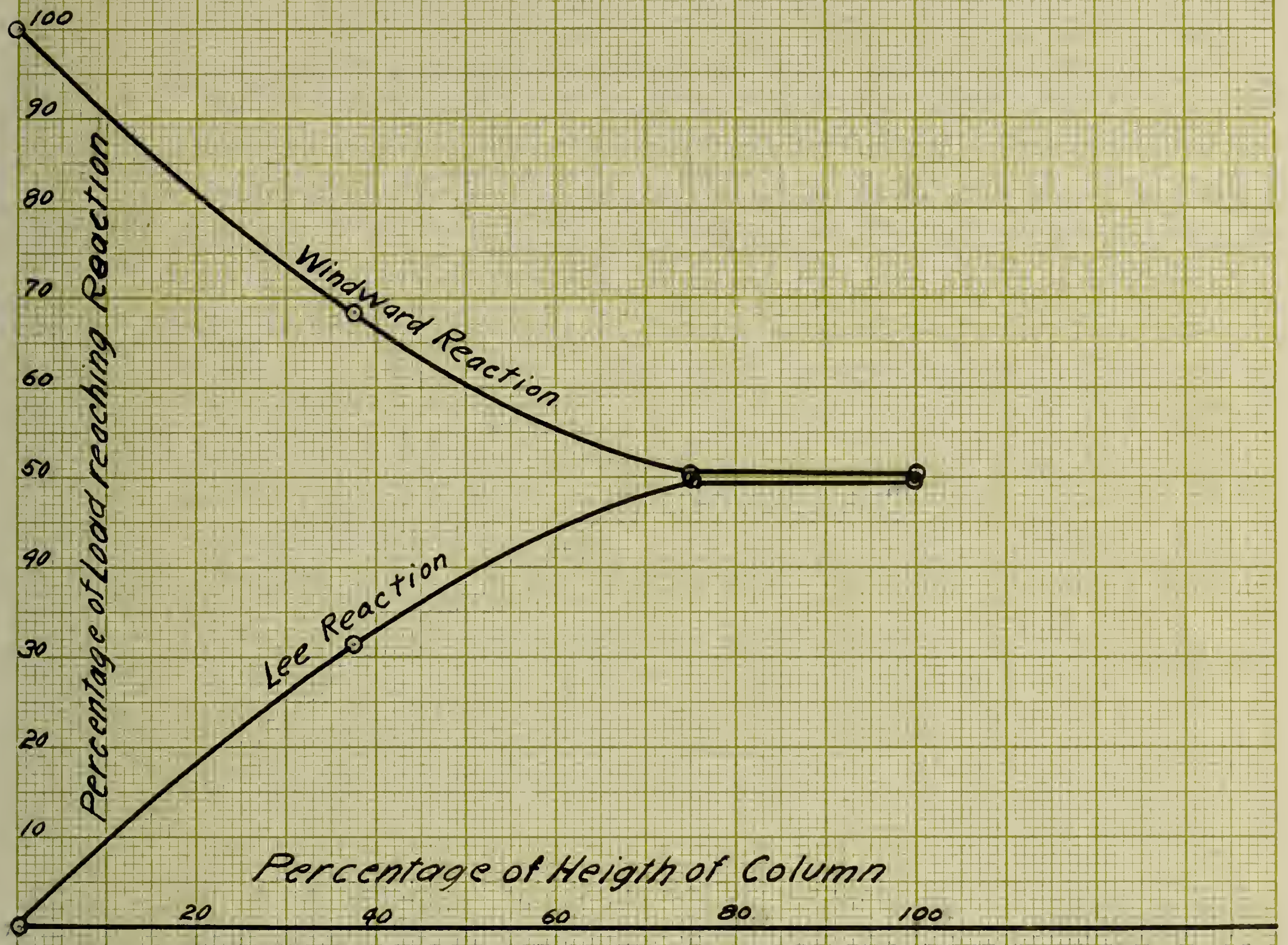


Fig. 32
Curves showing
Variation of
Percentage of Load reaching each Reaction
as
Heigth of Point of Application from Base of Column Varies

(Span of Bent is kept constant)